

Deterministic Near-Linear Time Minimum Cut for Weighted Graphs

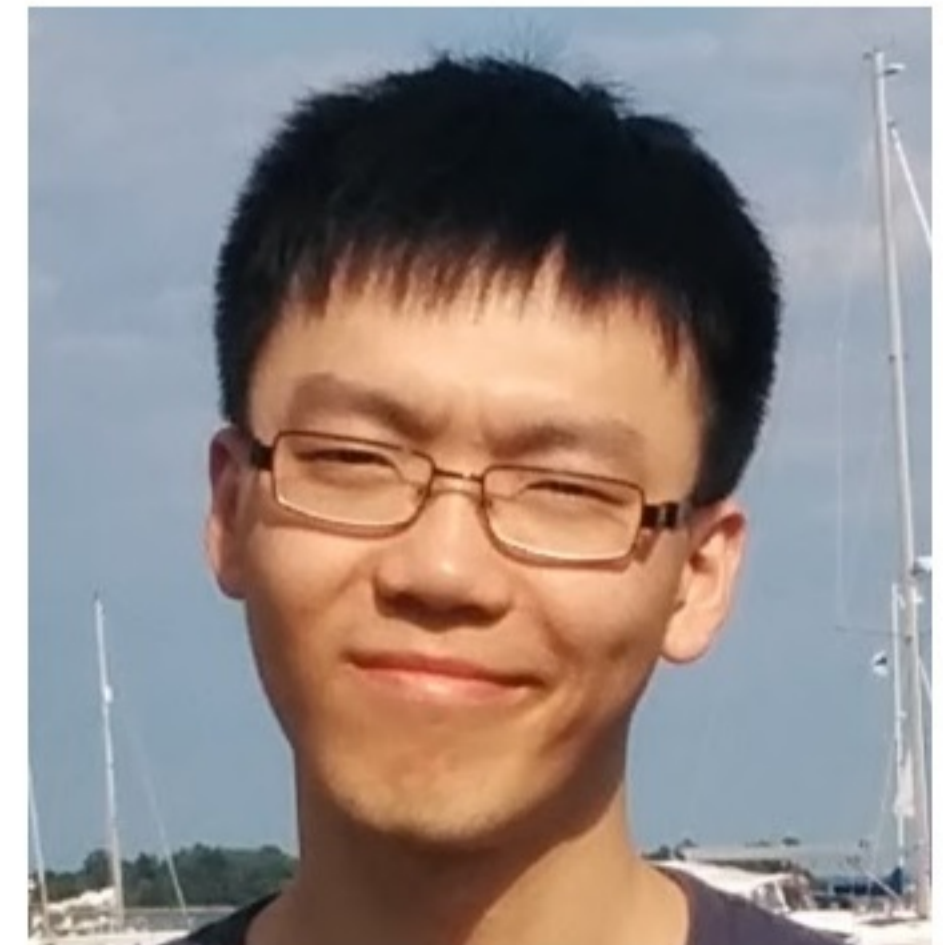
Jason Li (Berkeley→CMU)



with **Monika Henzinger**



Satish Rao



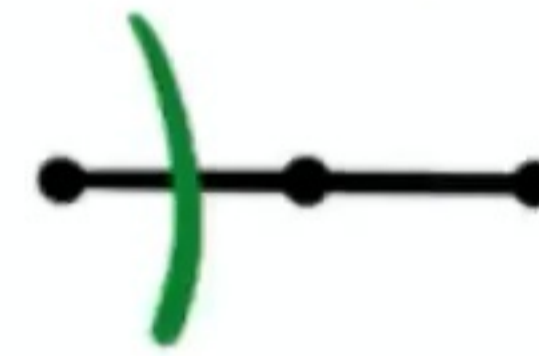
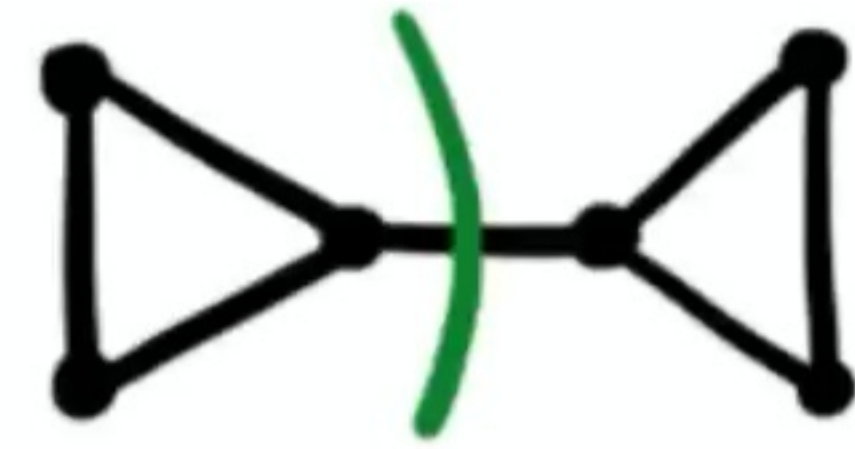
Di Wang

Introduction

- **Global mincut problem:**
given a weighted undirected graph,
delete edges of minimum weight
to disconnect the graph

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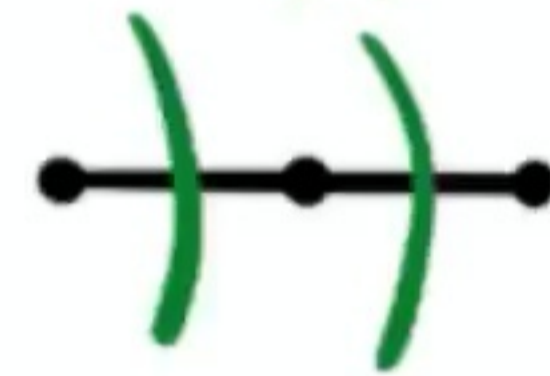
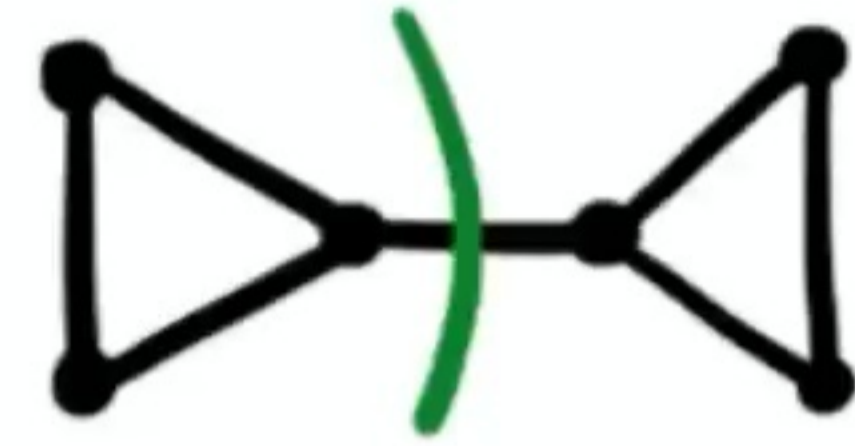
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all edges
weight 1

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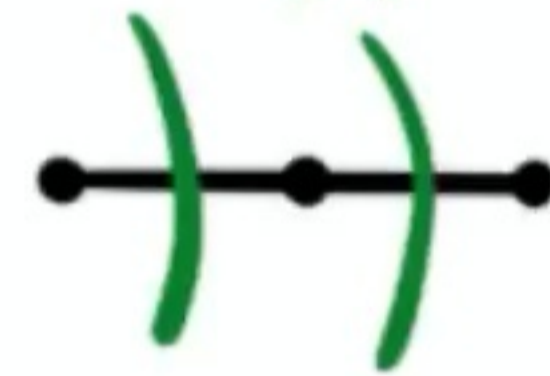
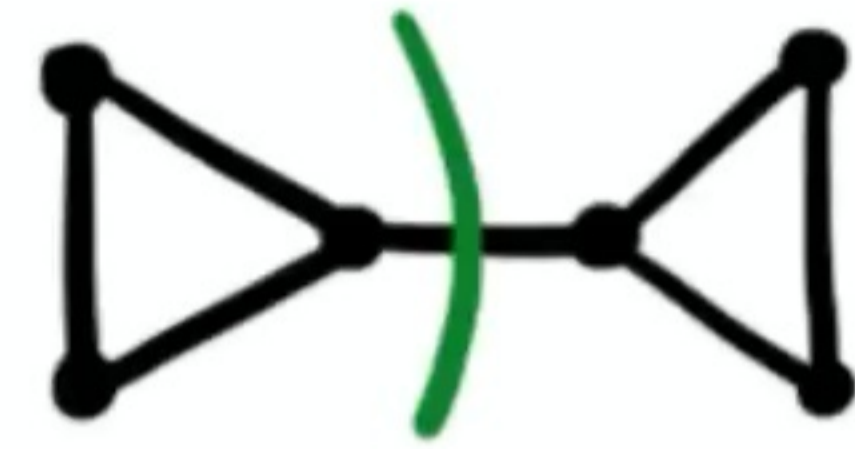
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- [Karger'96] randomized algorithm in $O(m \log^3 n)$ time
"Is there a **deterministic** near-linear time algorithm?"

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"almost-linear time"
- [This work] $\tilde{O}(m)$ time, answering Karger's question

Outline

- **Local win/win approach to global mincut**

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- Our main structural theorem

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- New randomized $(1+\varepsilon)$ -approximate global mincut
(main conceptual contribution)

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- Our main structural theorem
- New randomized $(1+\varepsilon)$ -approximate global mincut
(main conceptual contribution)
- Derandomize and obtain exact mincut
(technical: 40+ pages)

Local Method

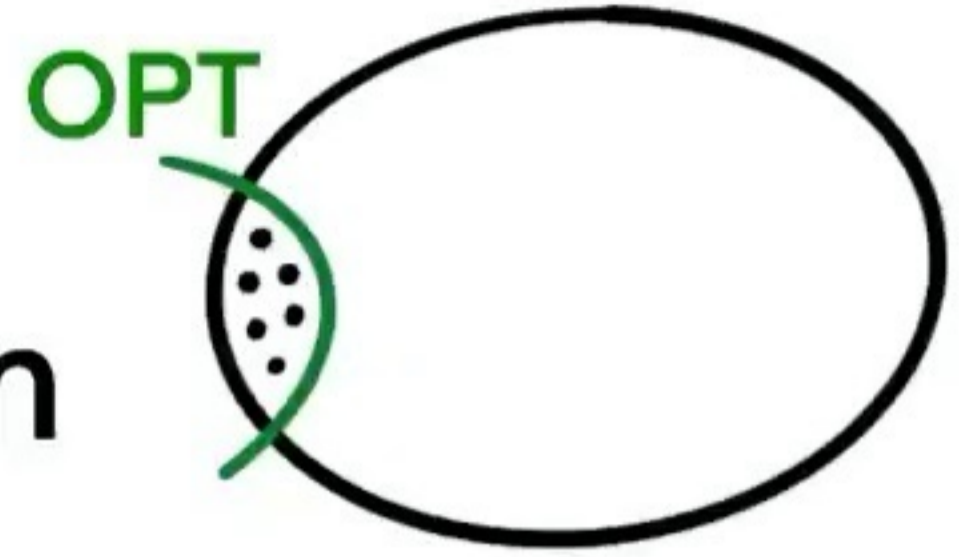
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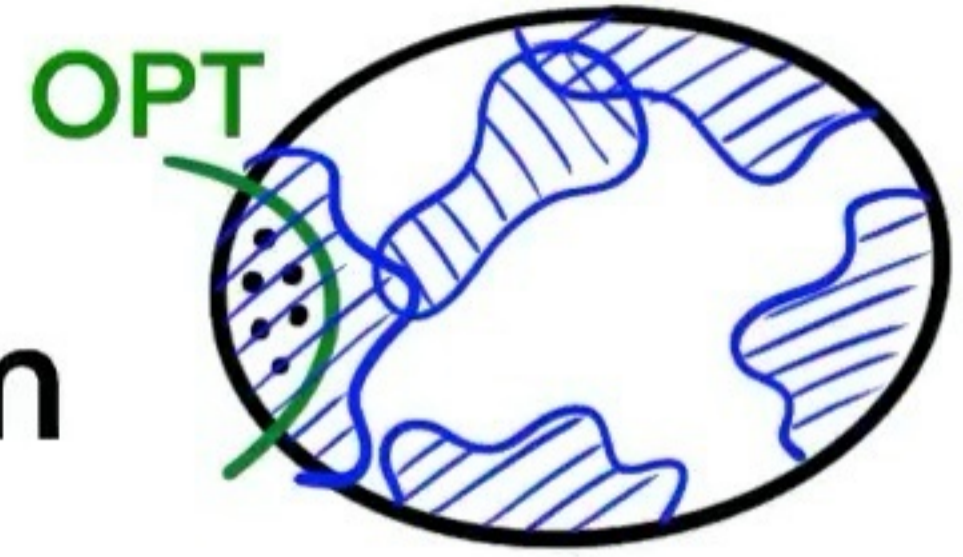
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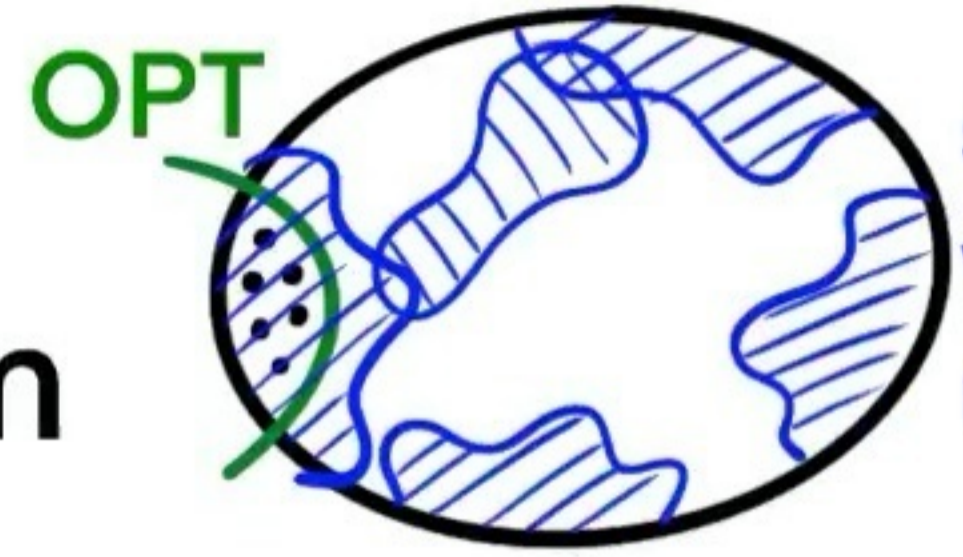
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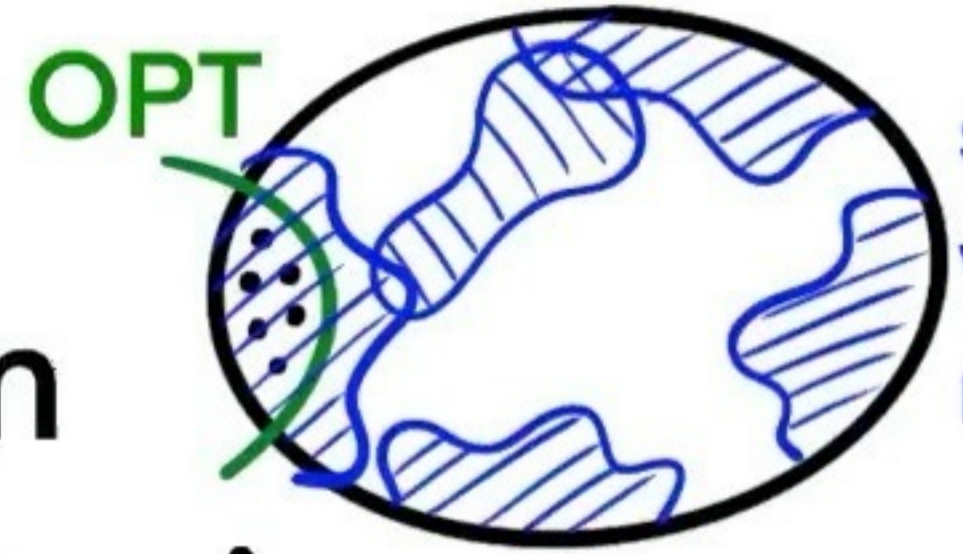


start at each
vertex:
 $n \times$ (local time)

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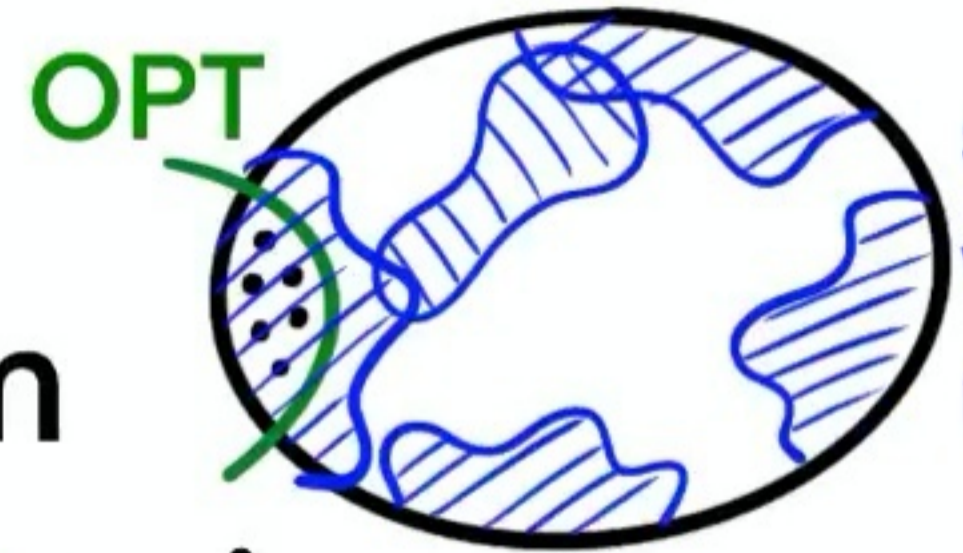


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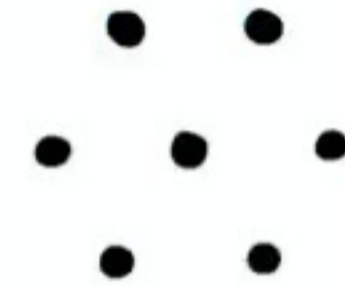
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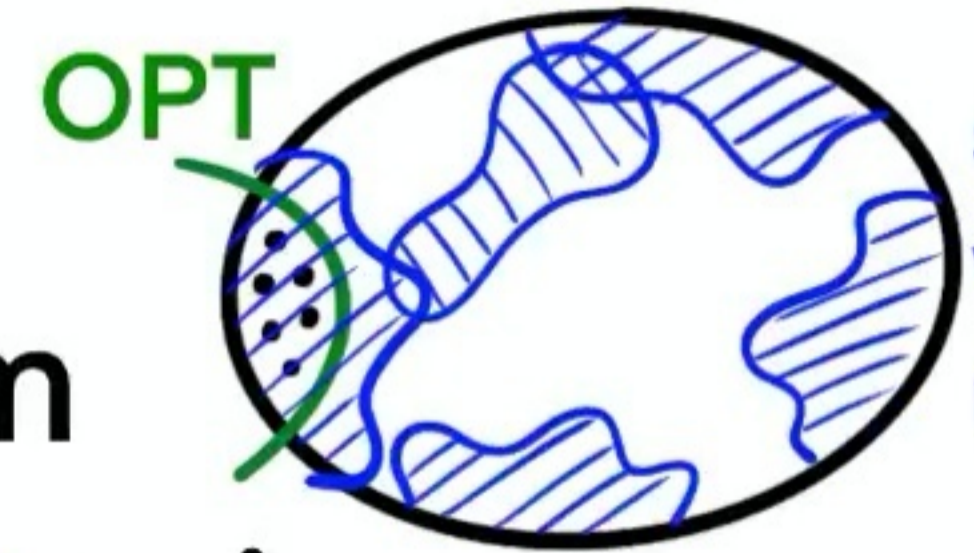
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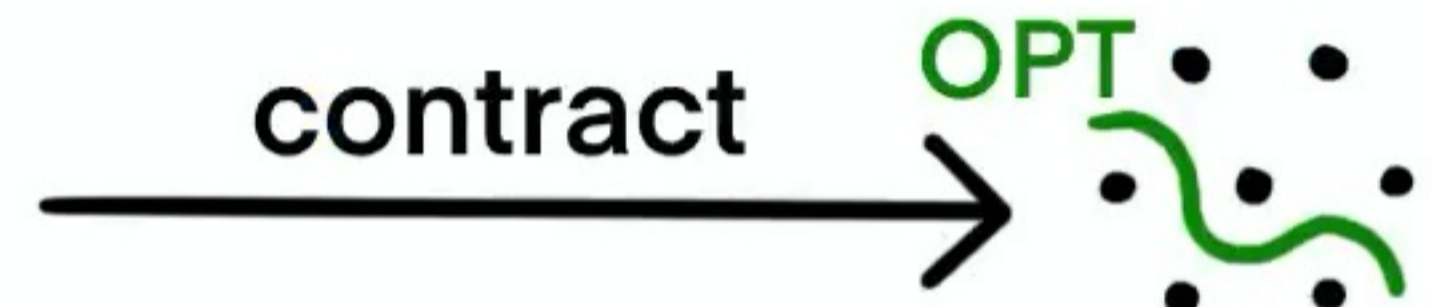
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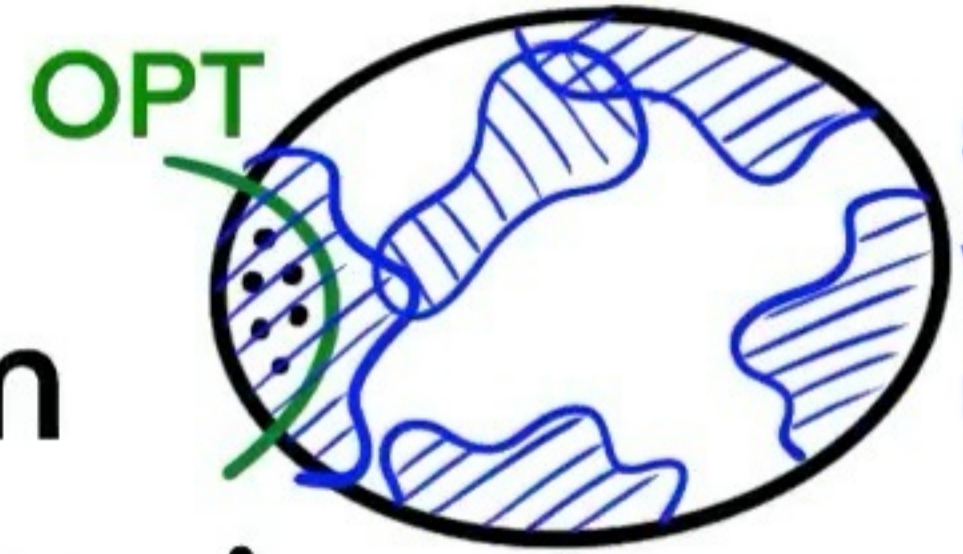
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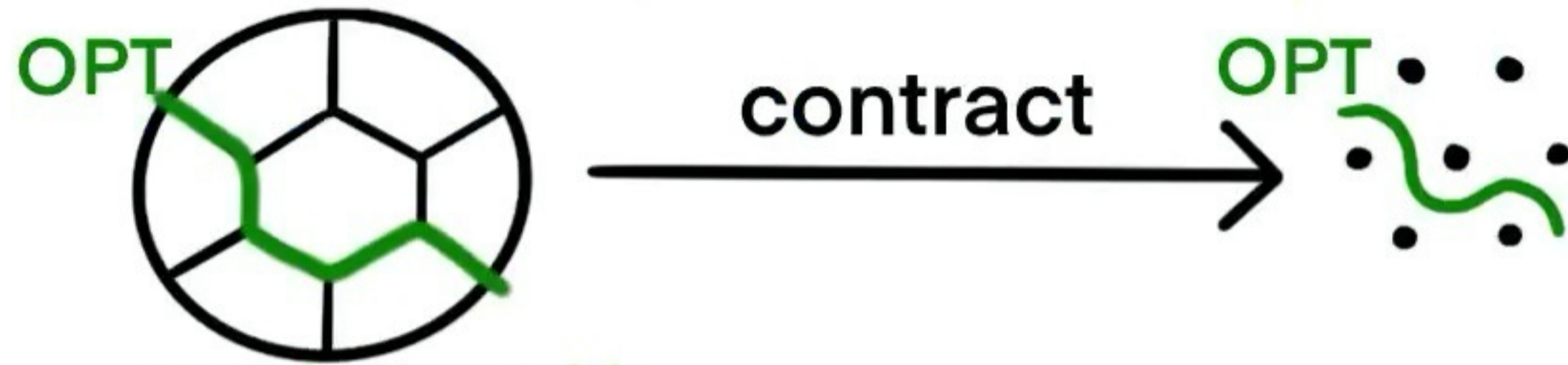
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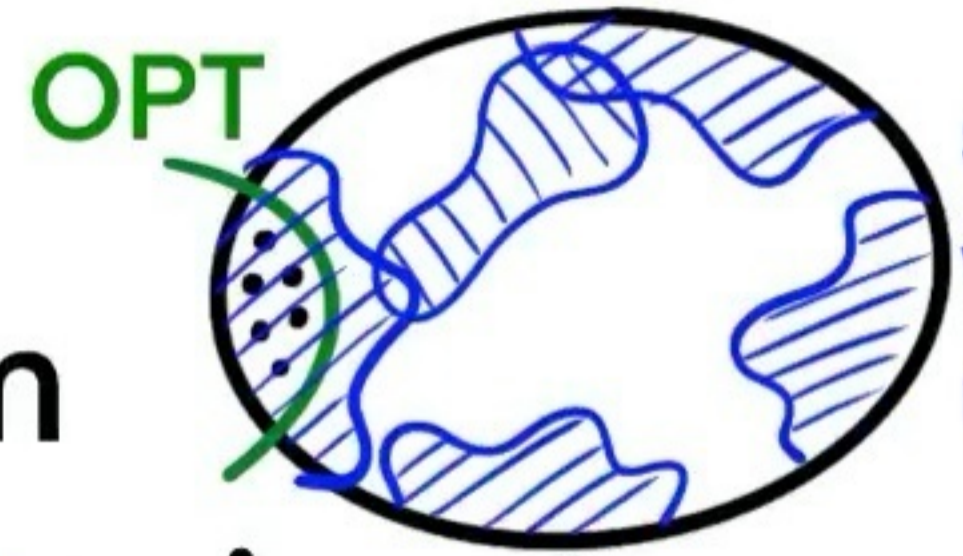
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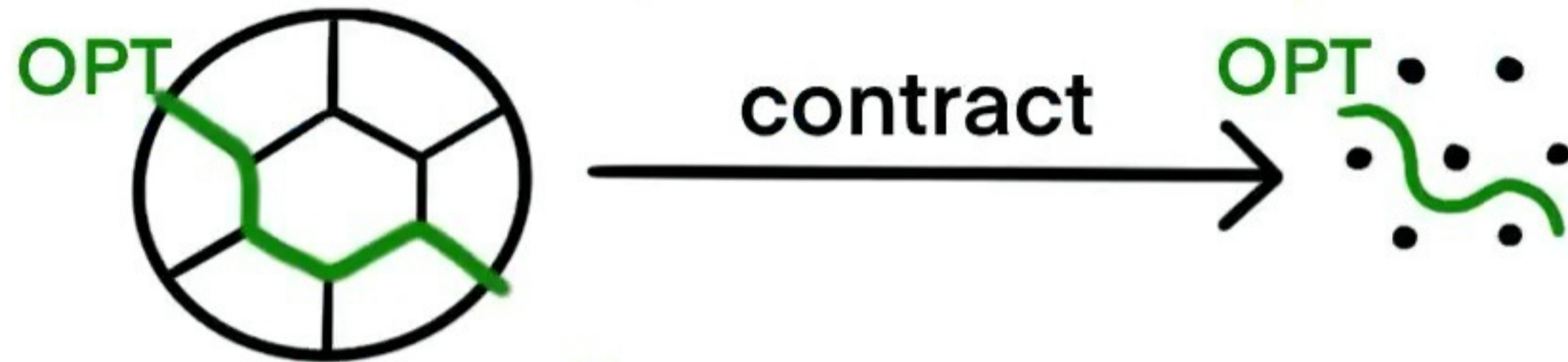
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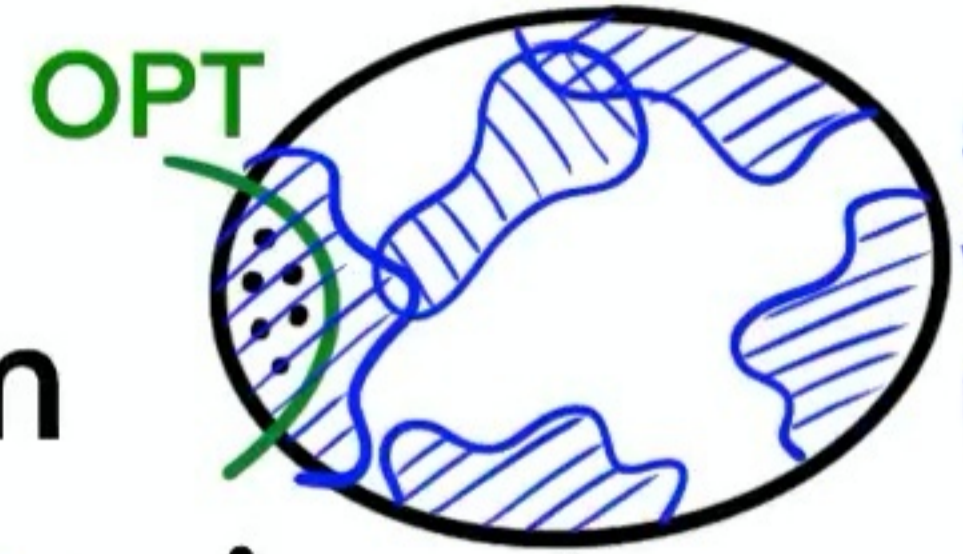
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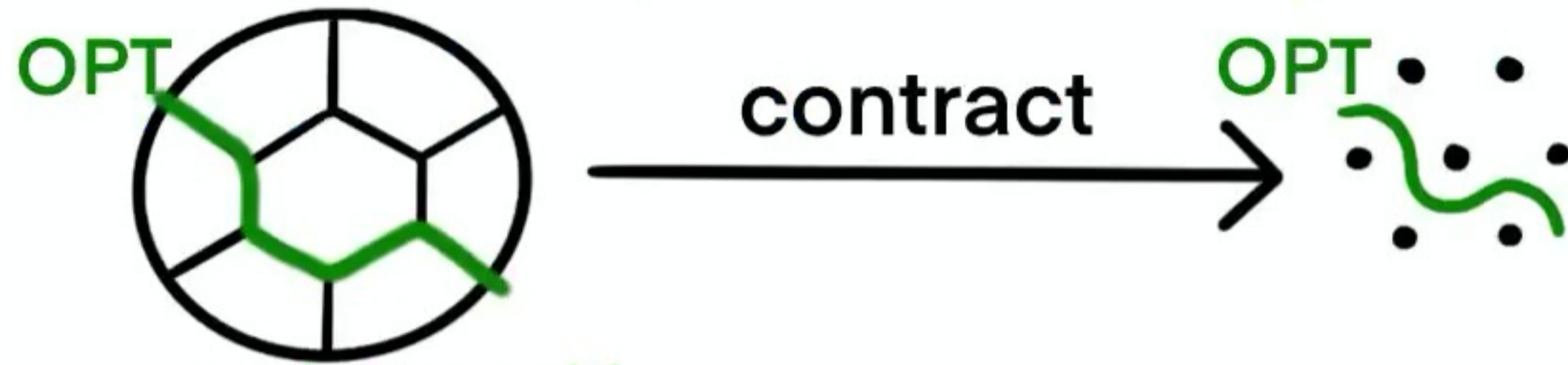
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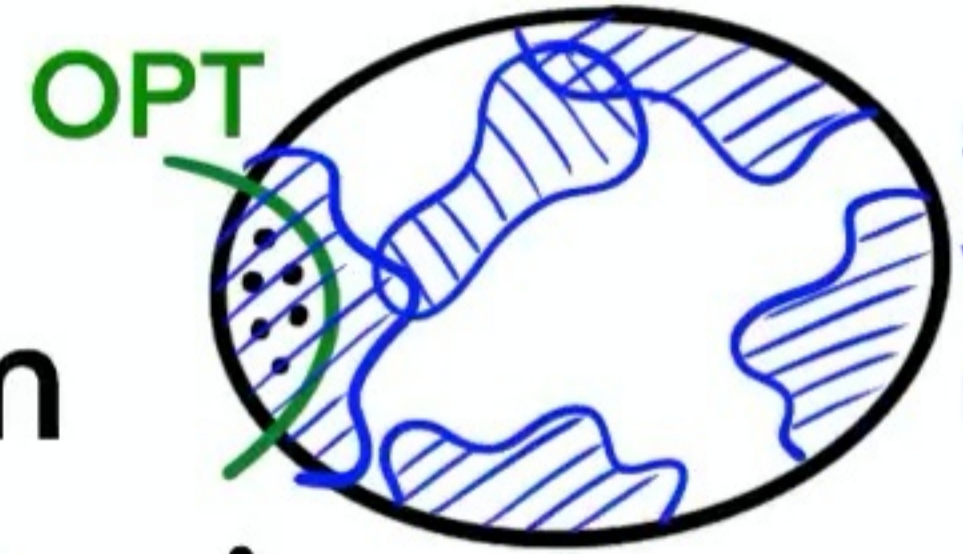
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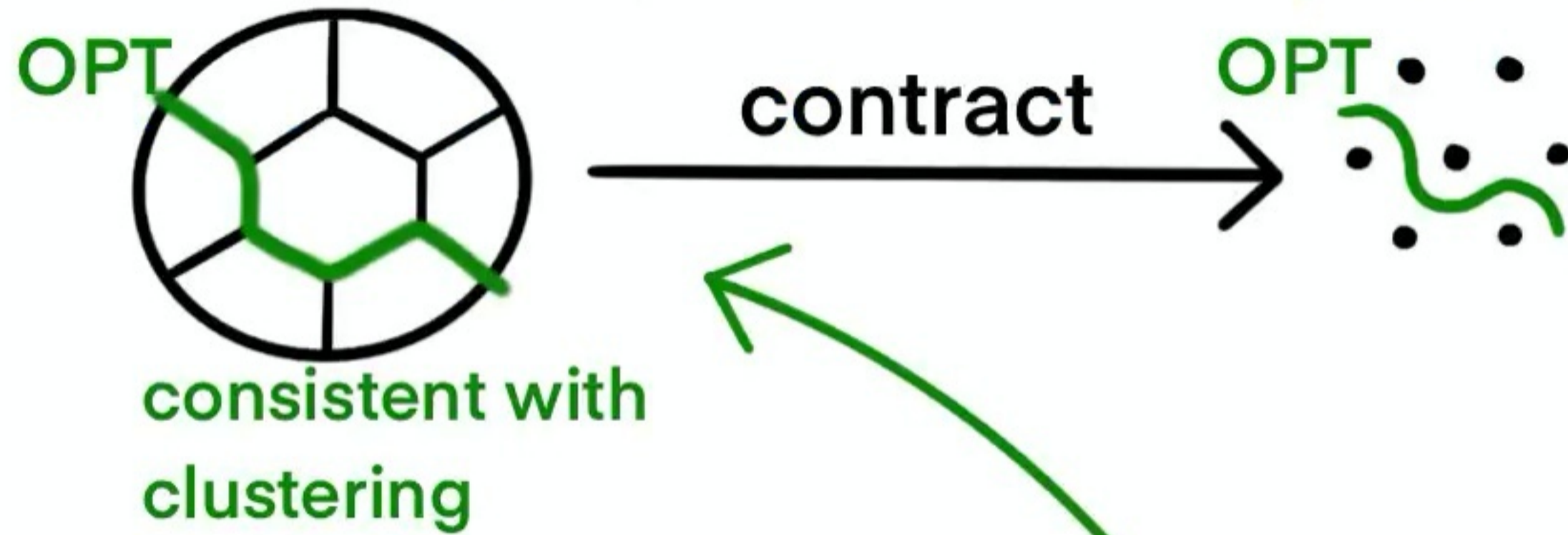
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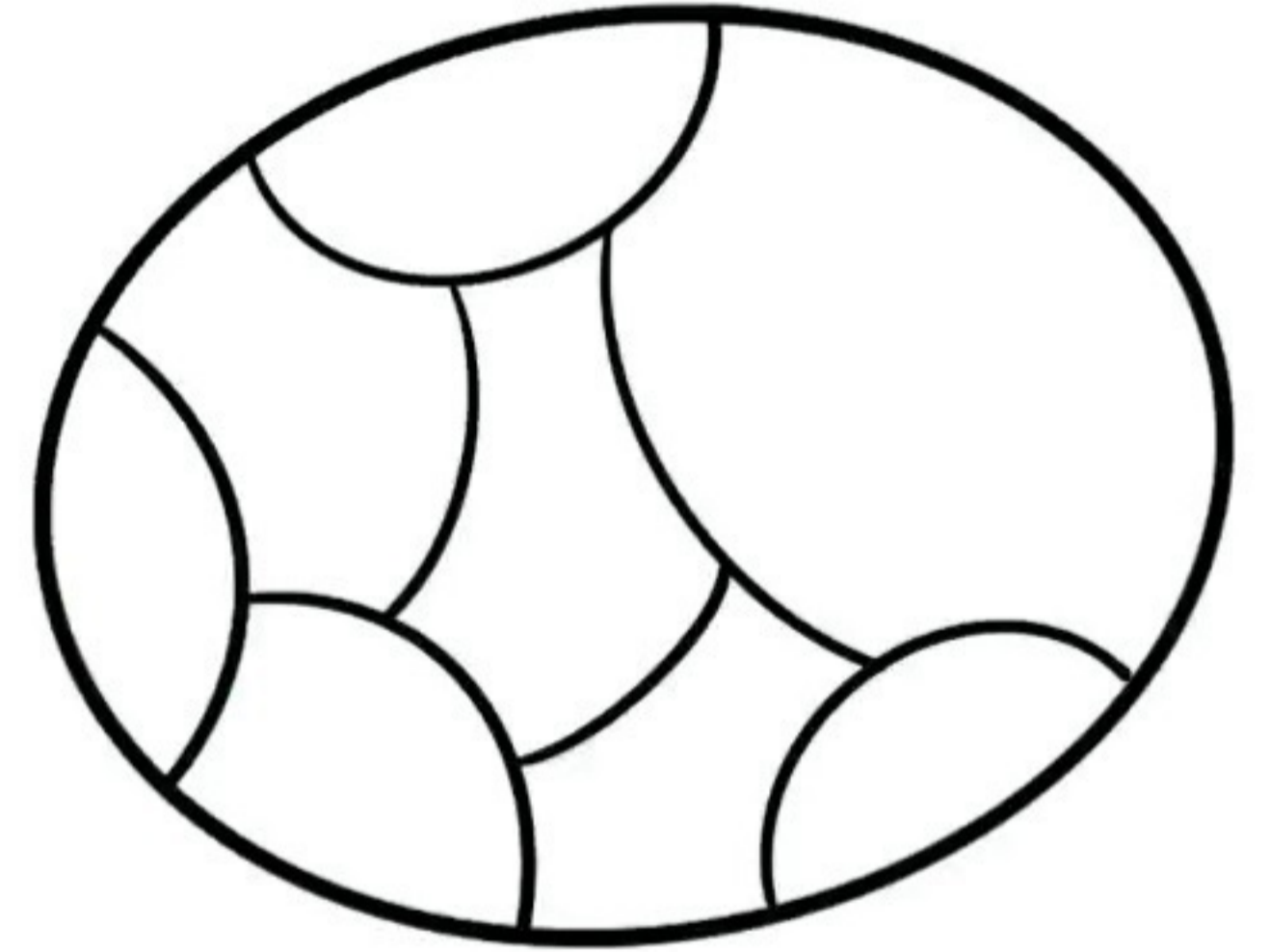
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Structural Theorem

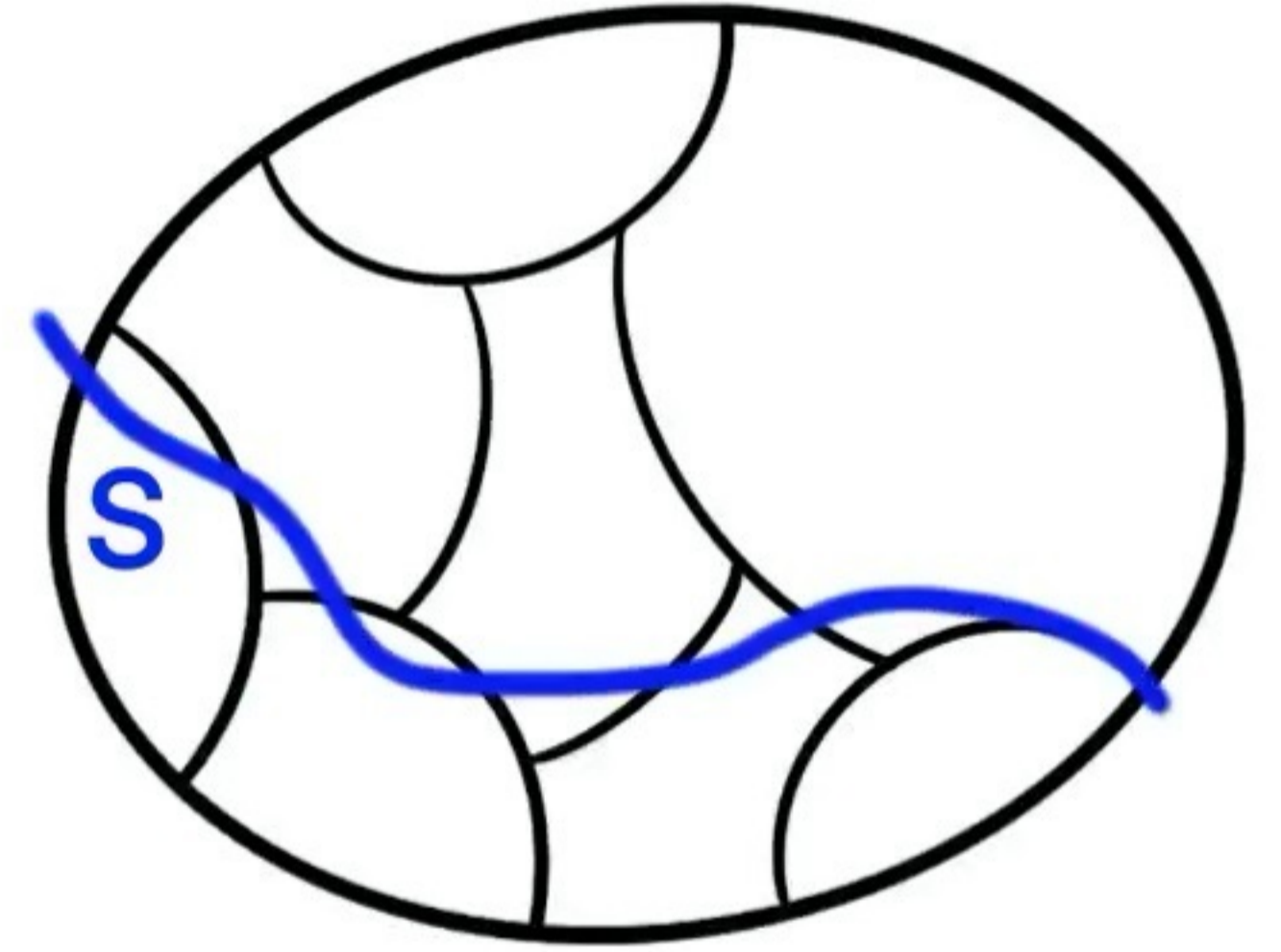
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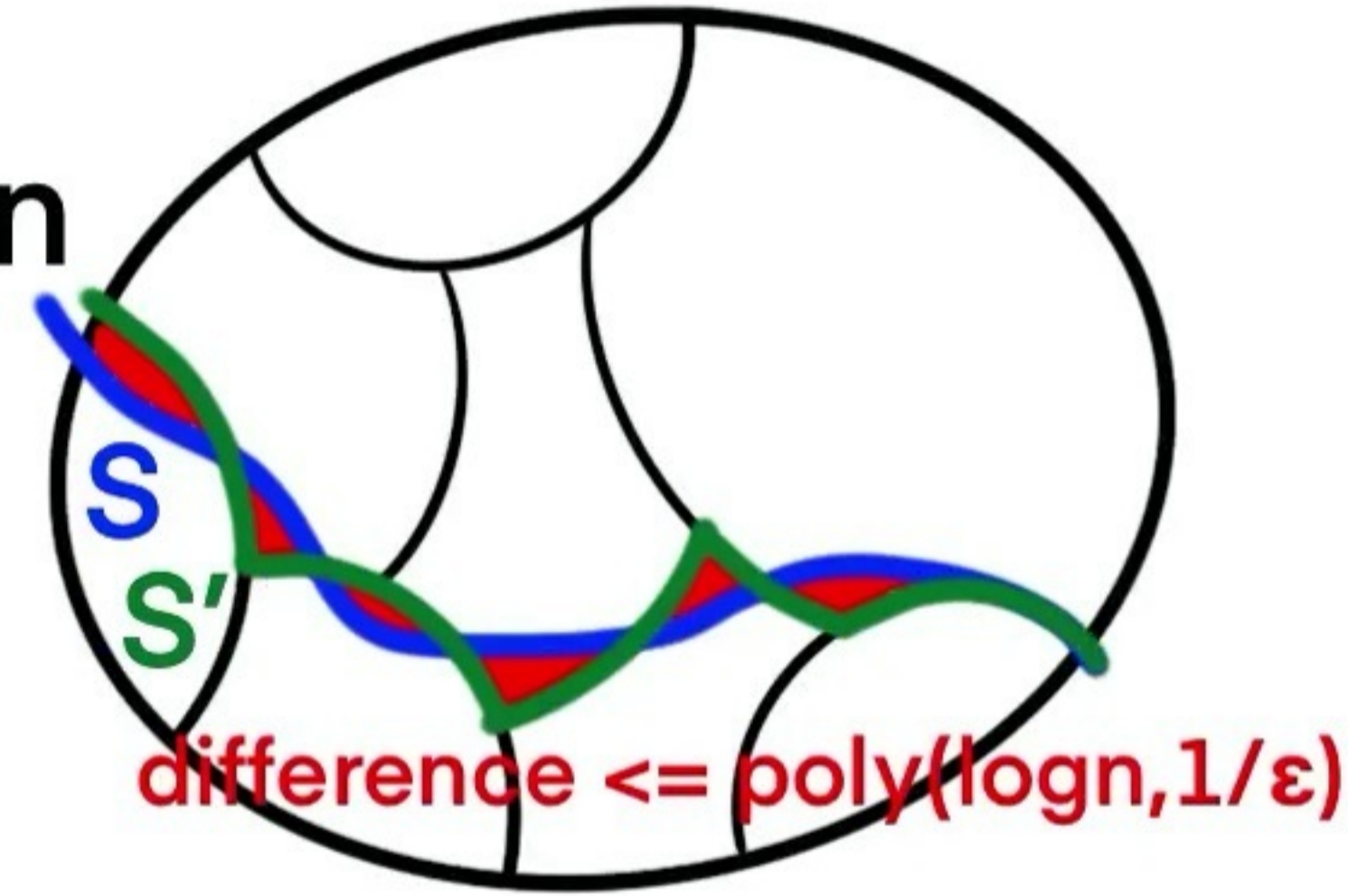
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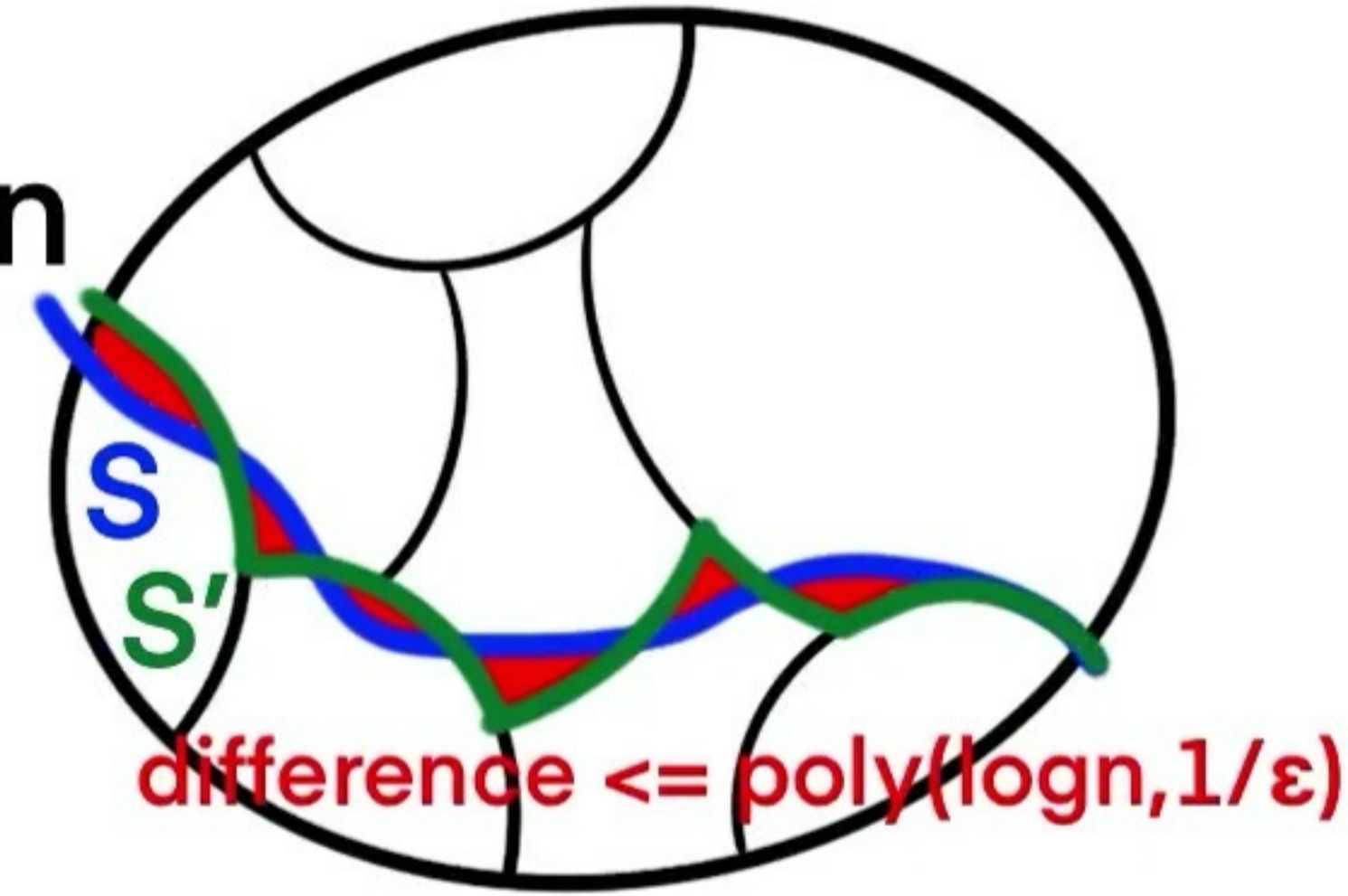
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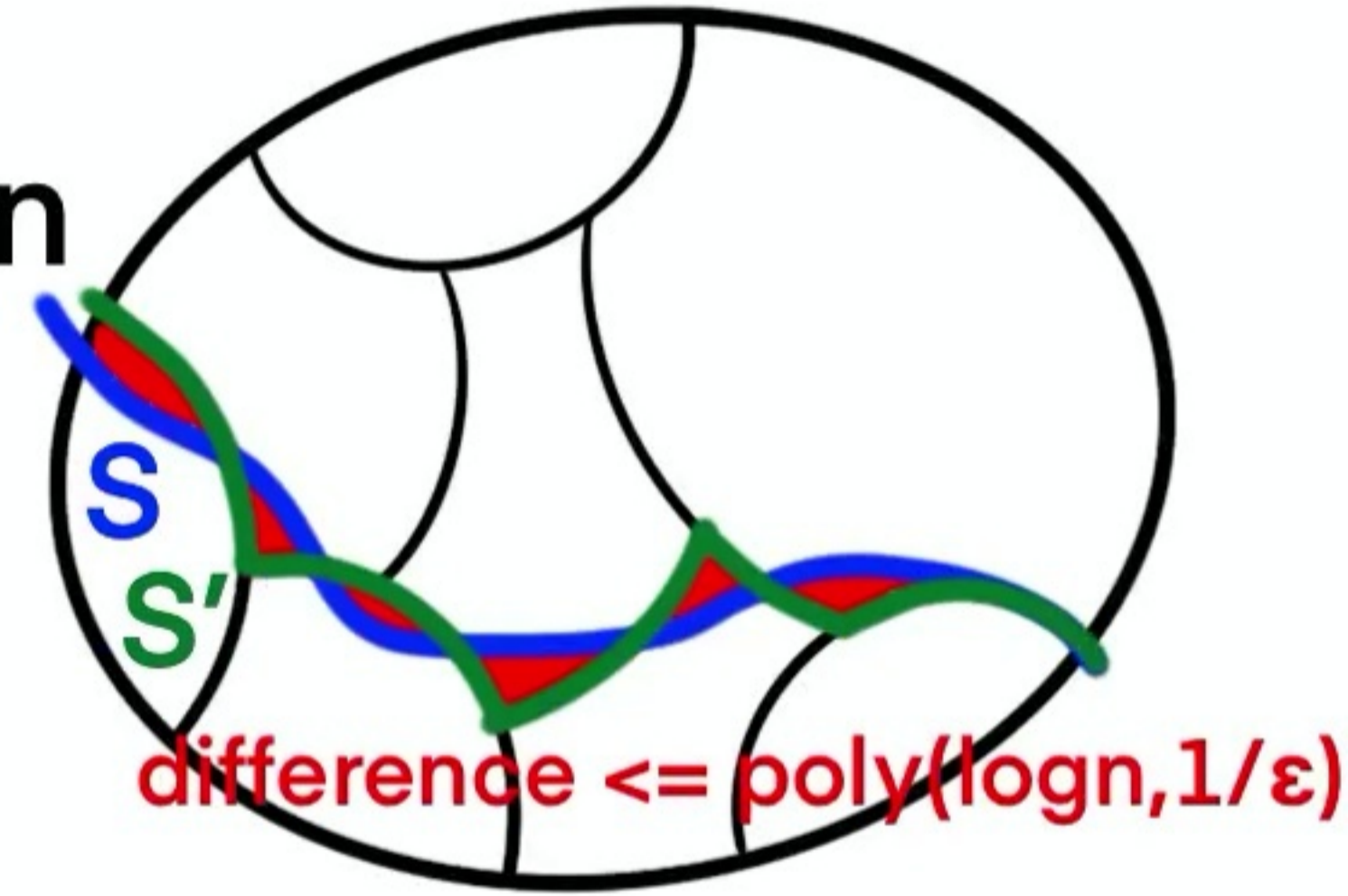
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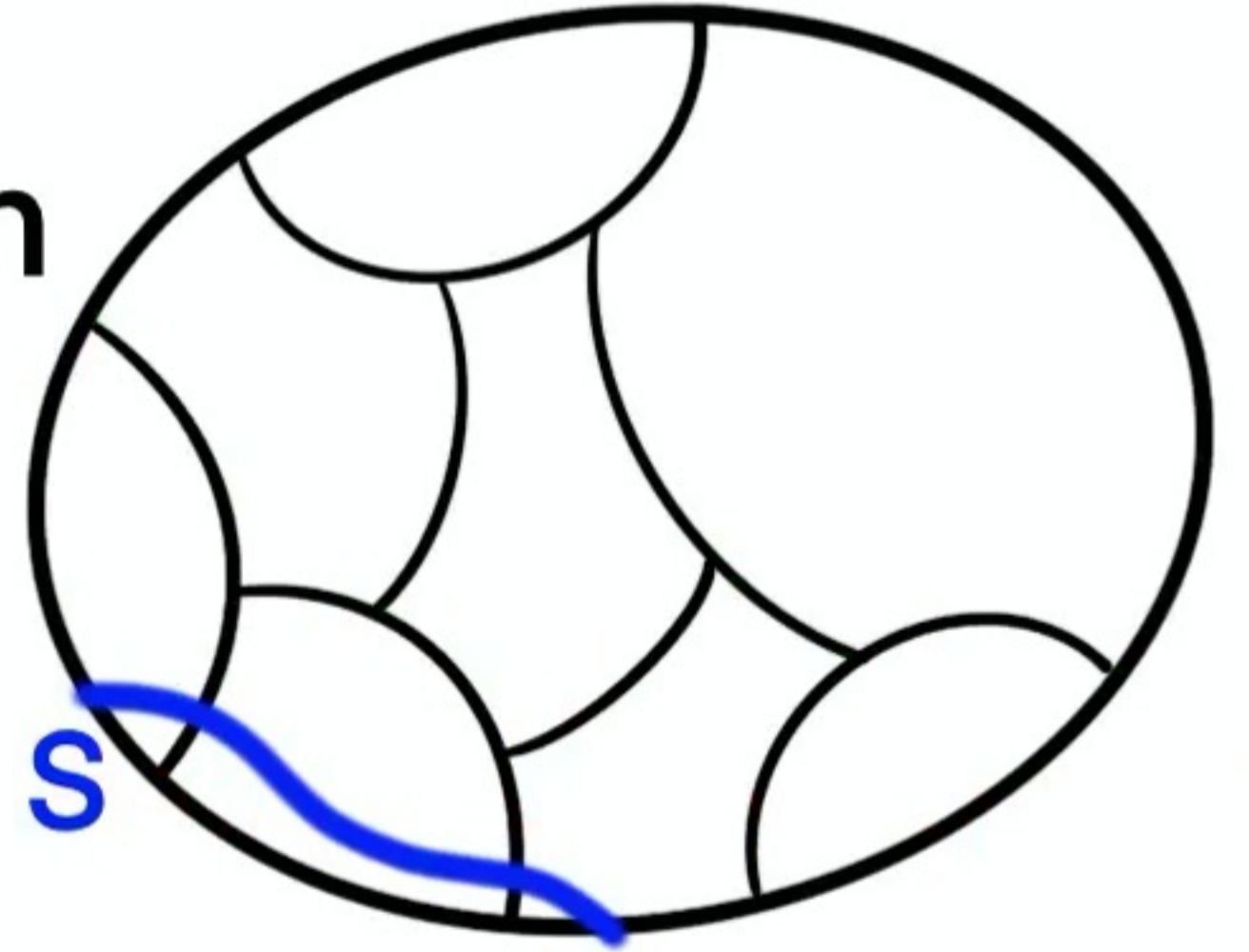
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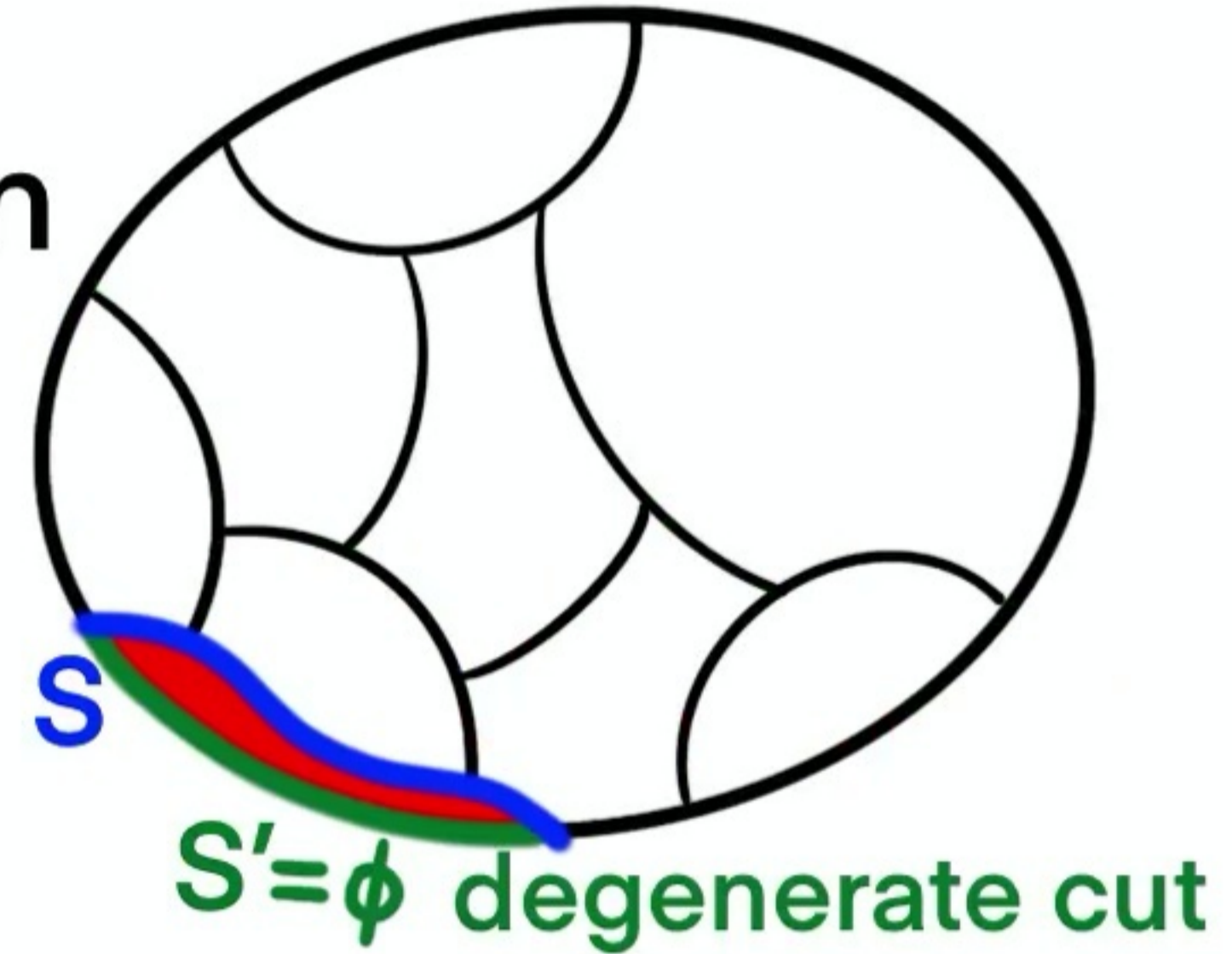
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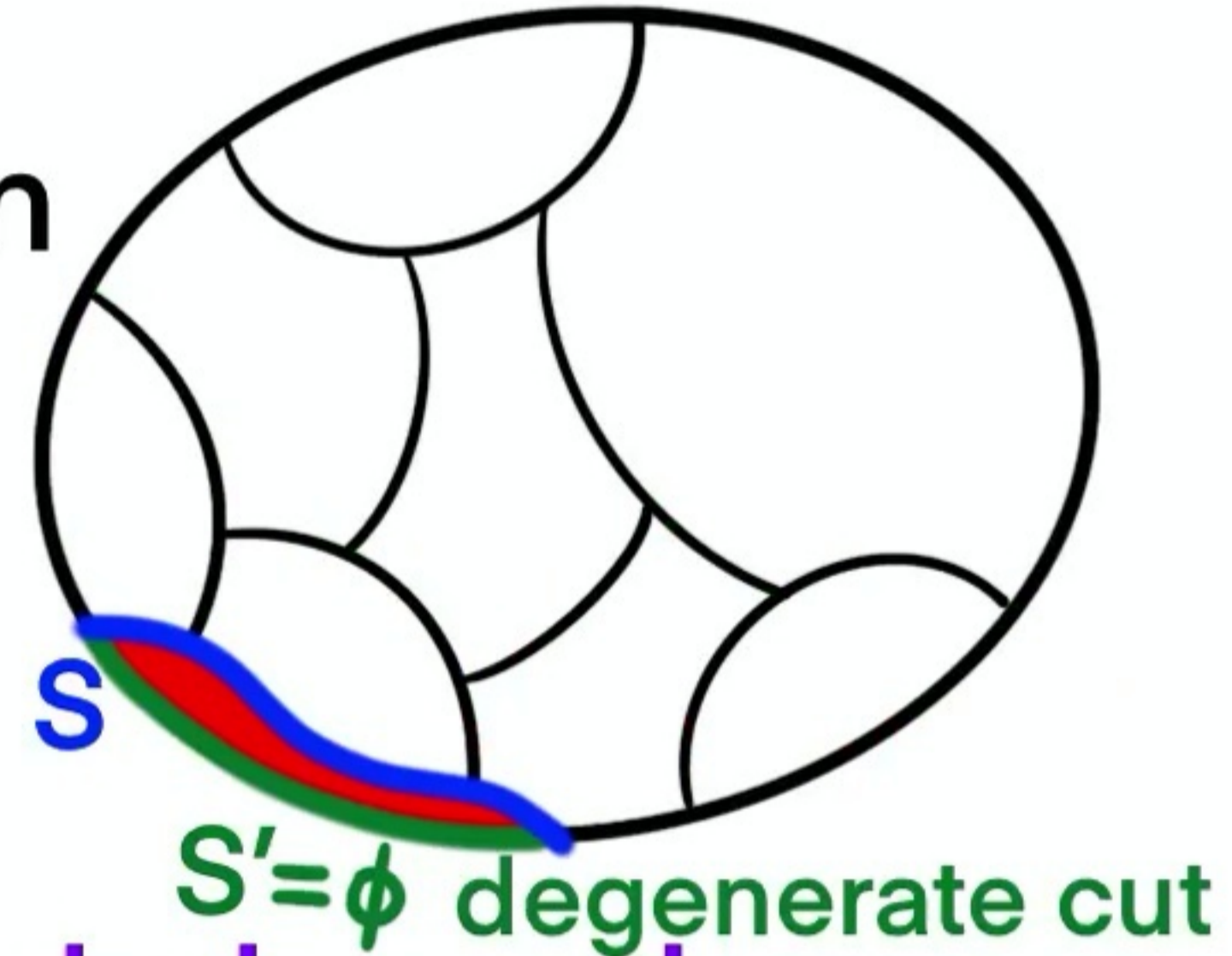
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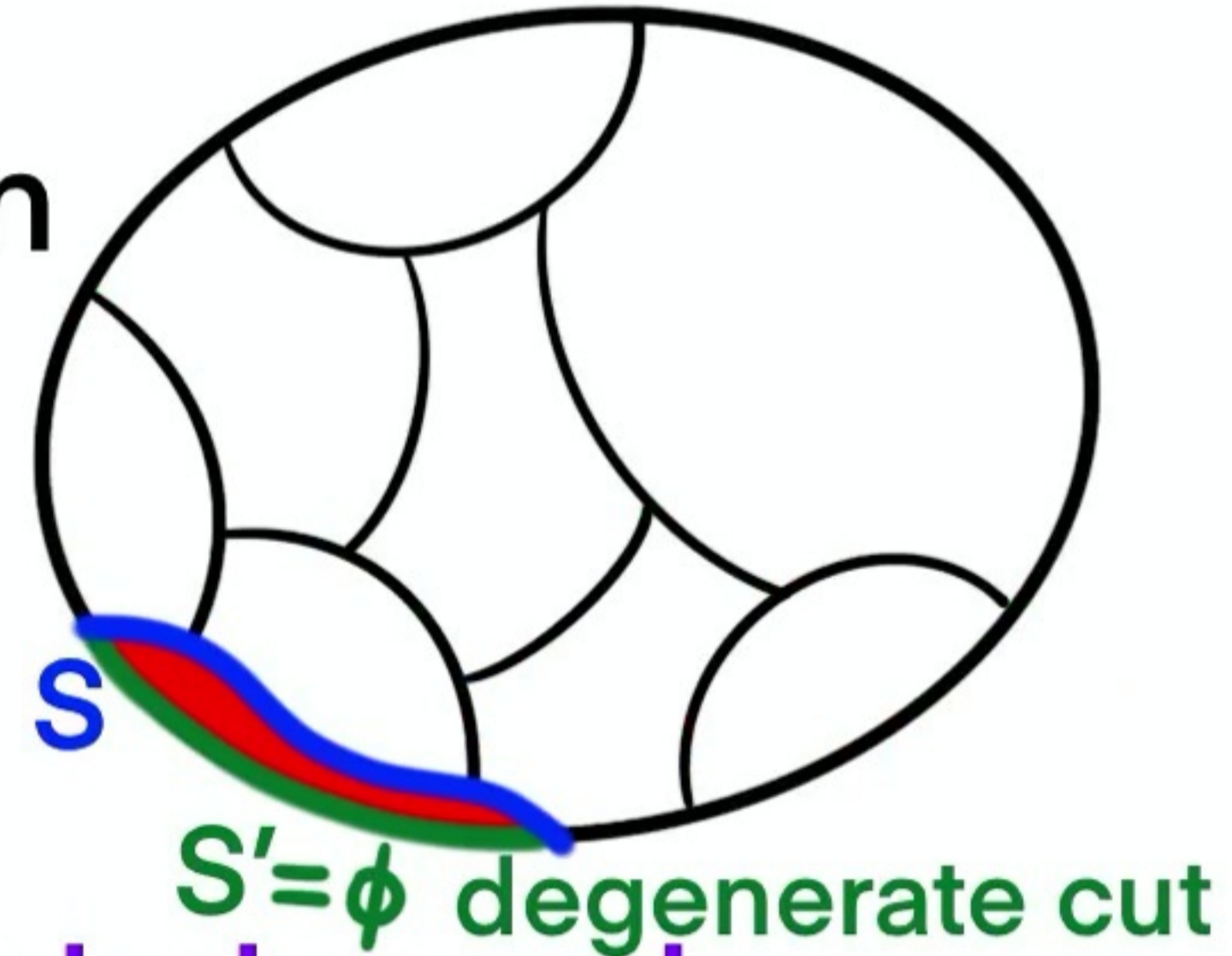
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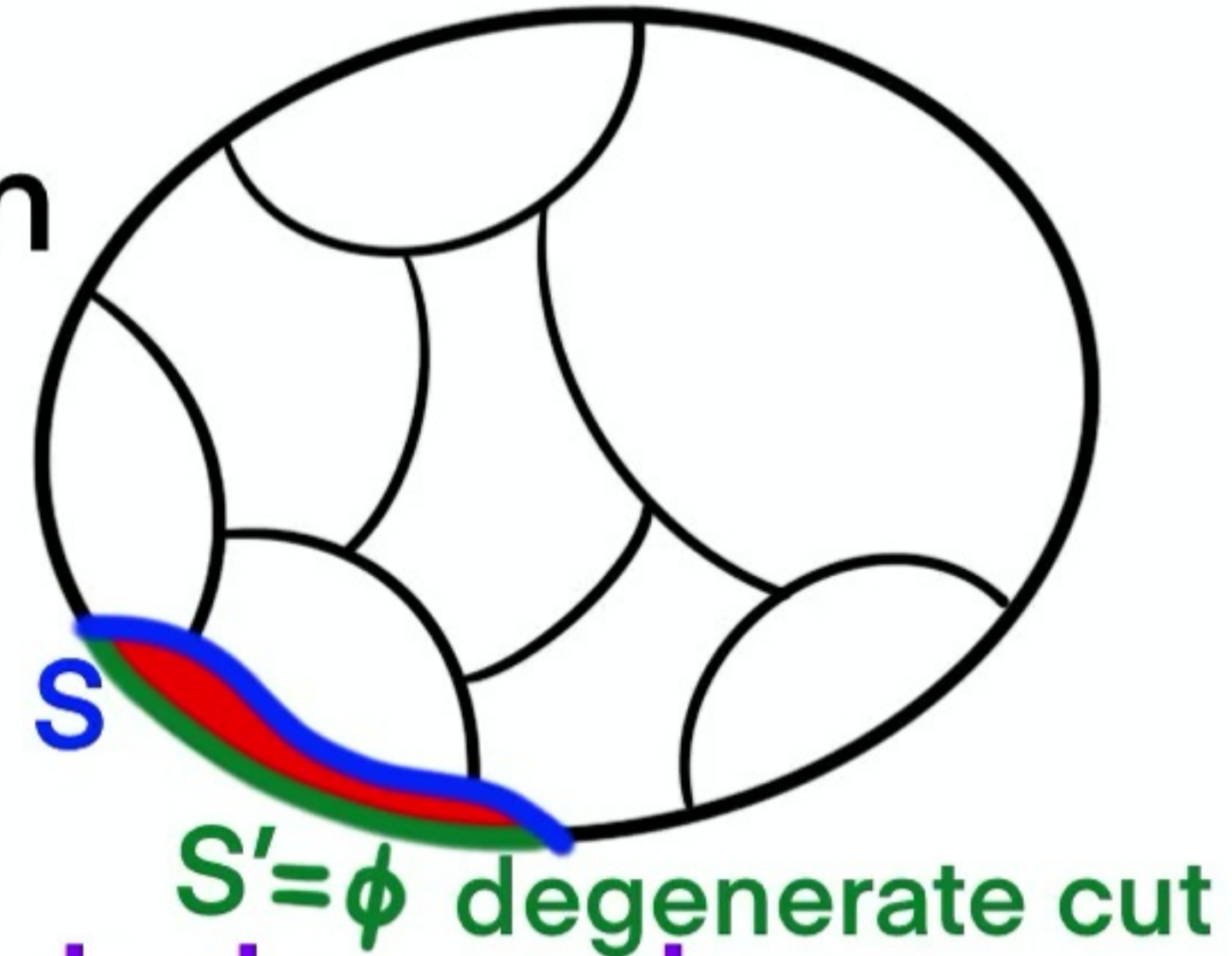
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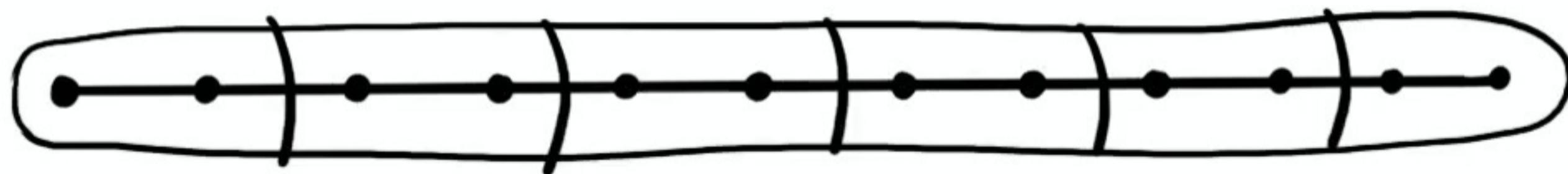
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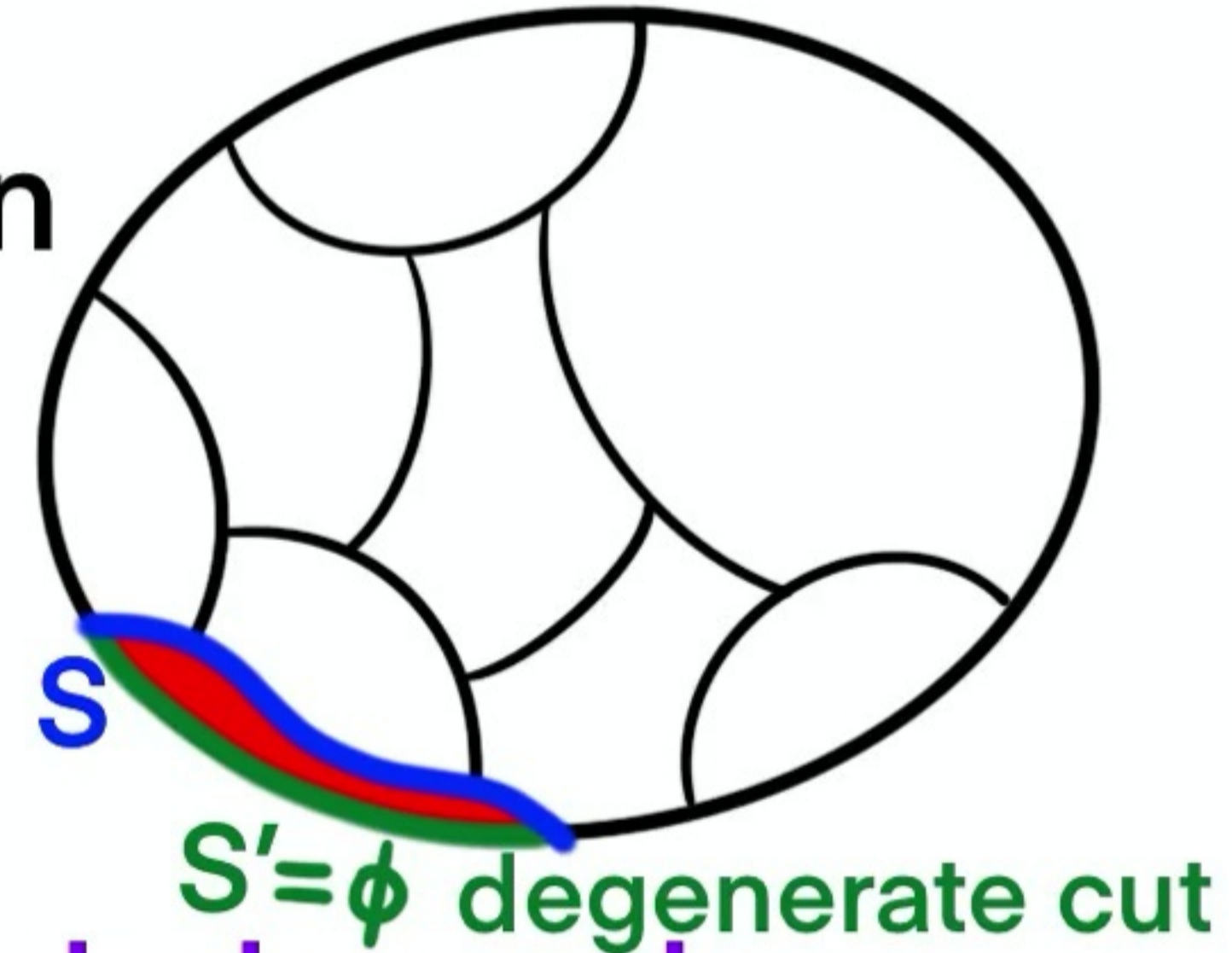
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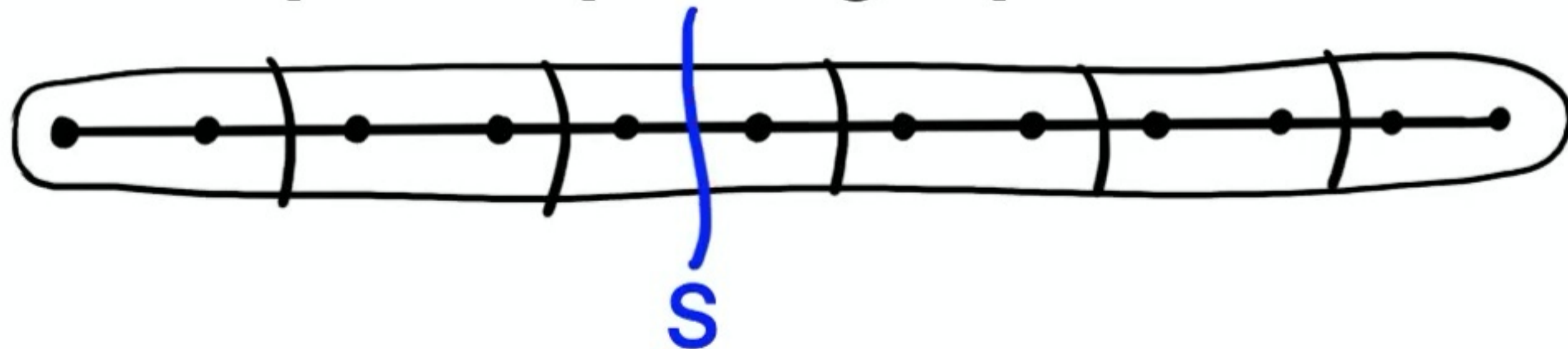
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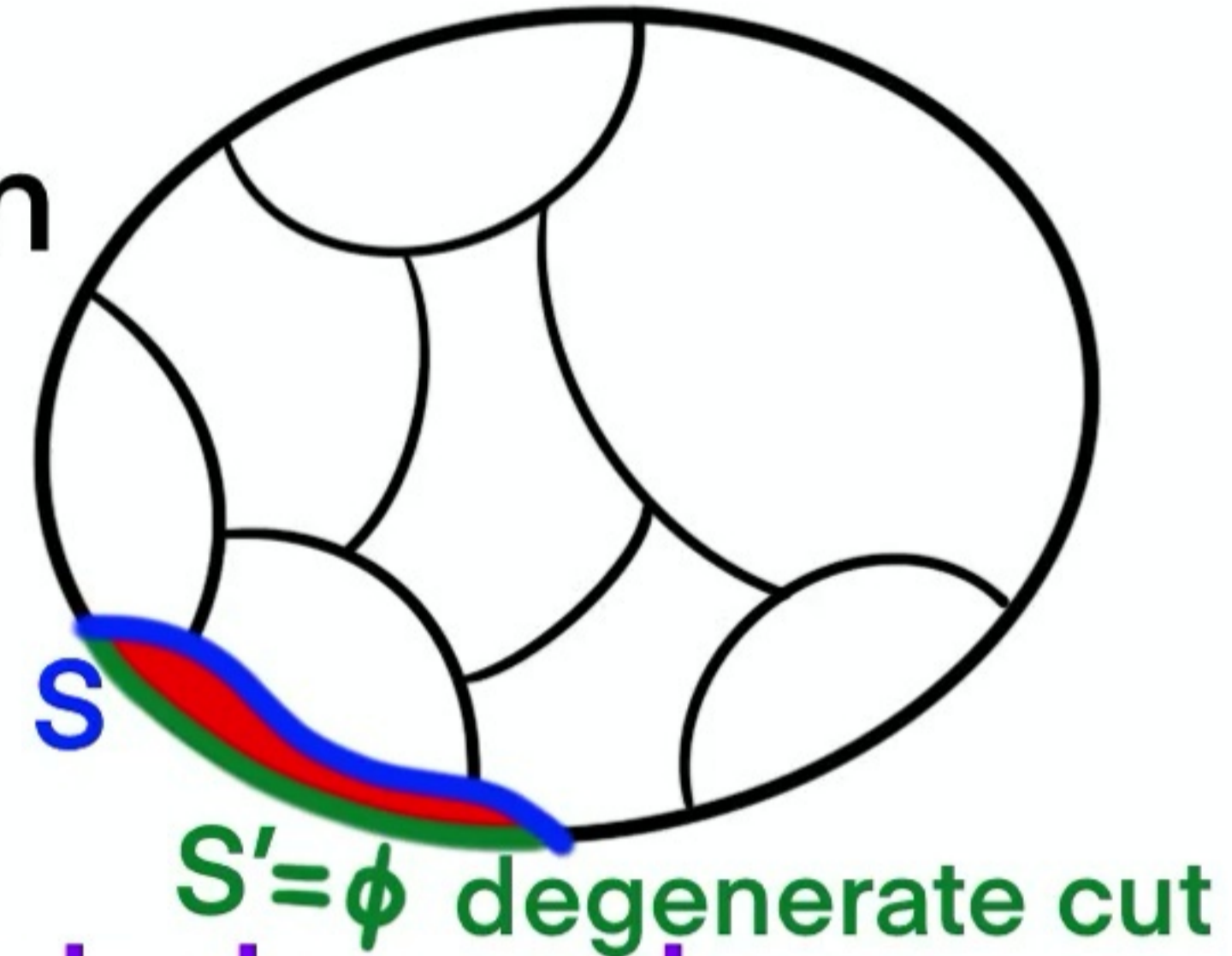
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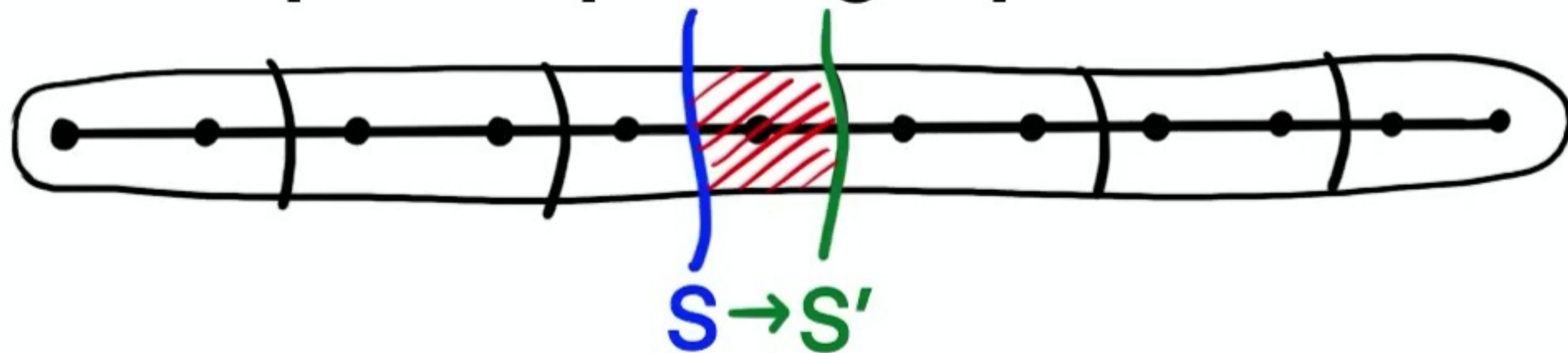
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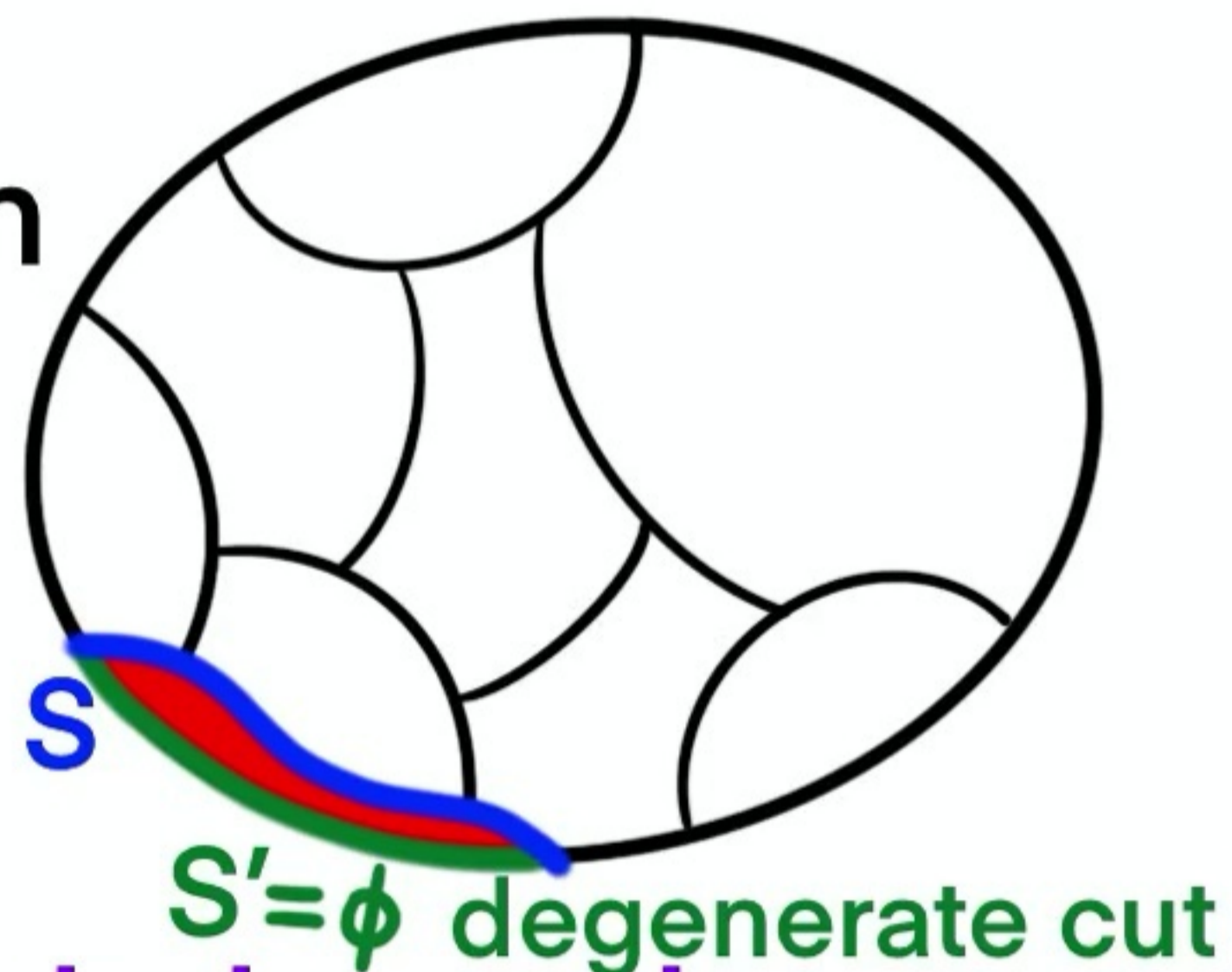
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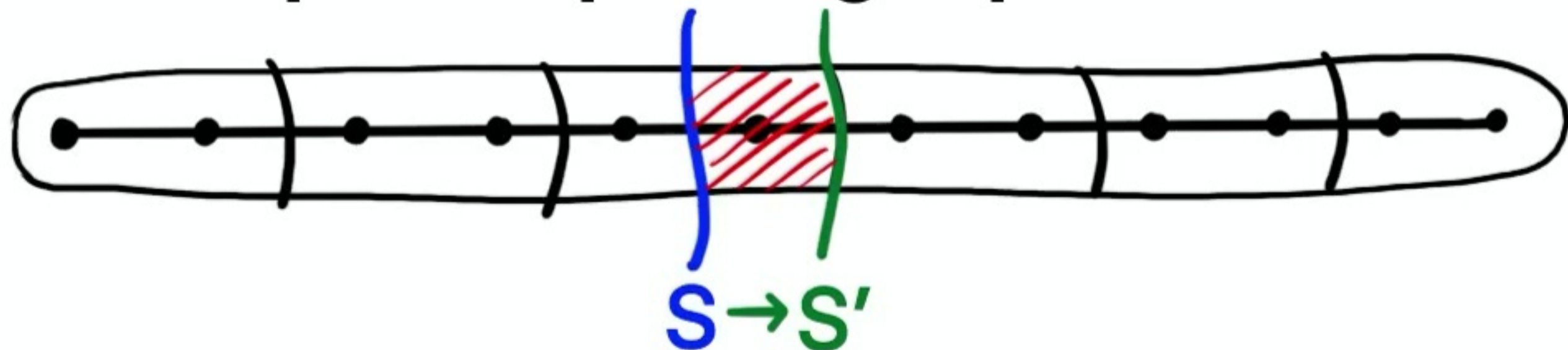
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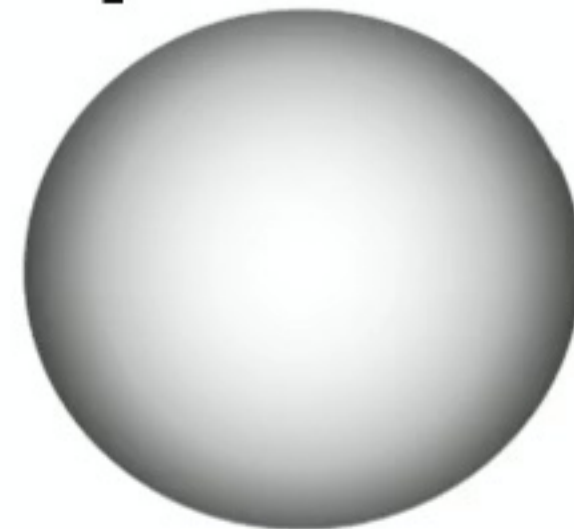
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Example 2: clique



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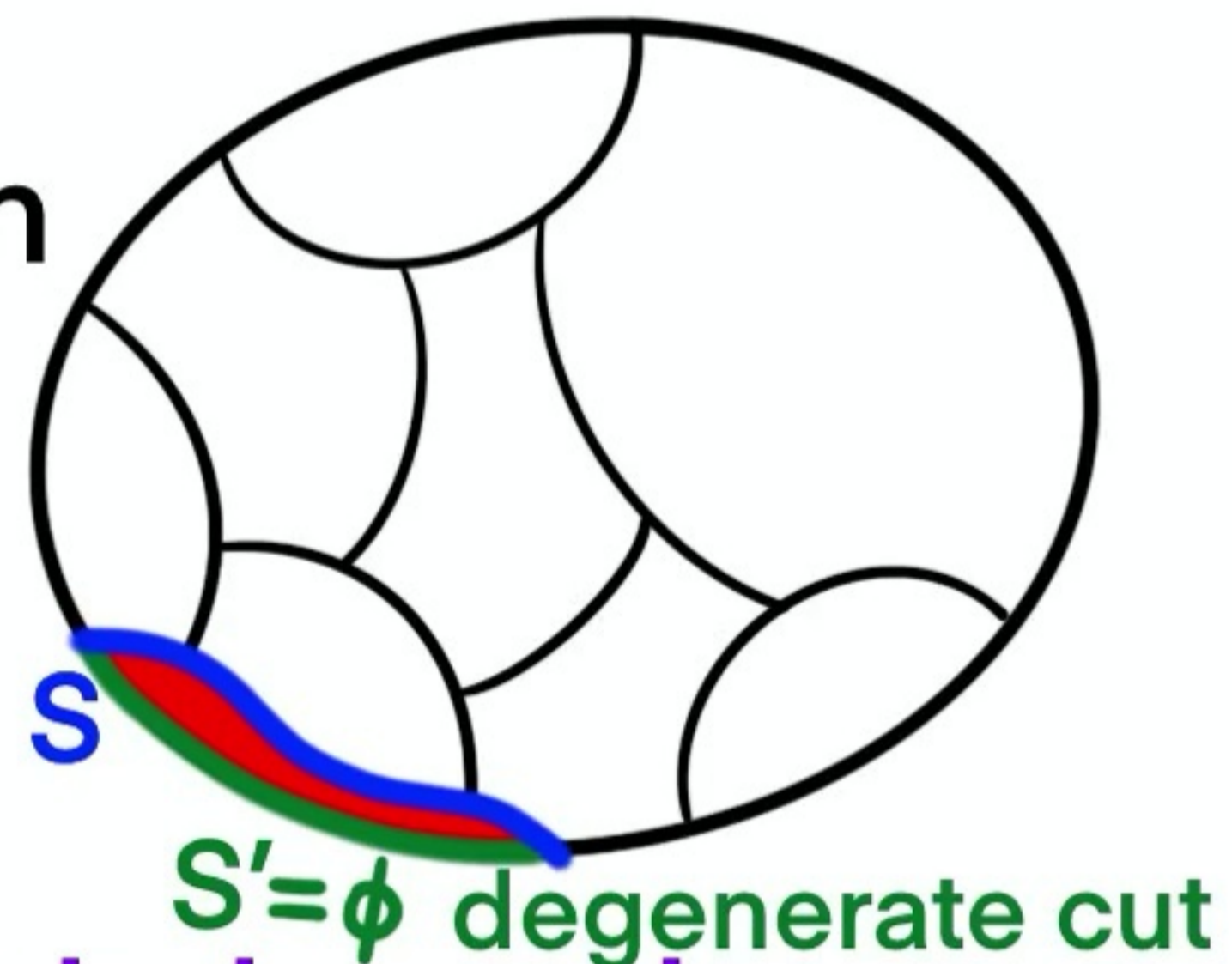
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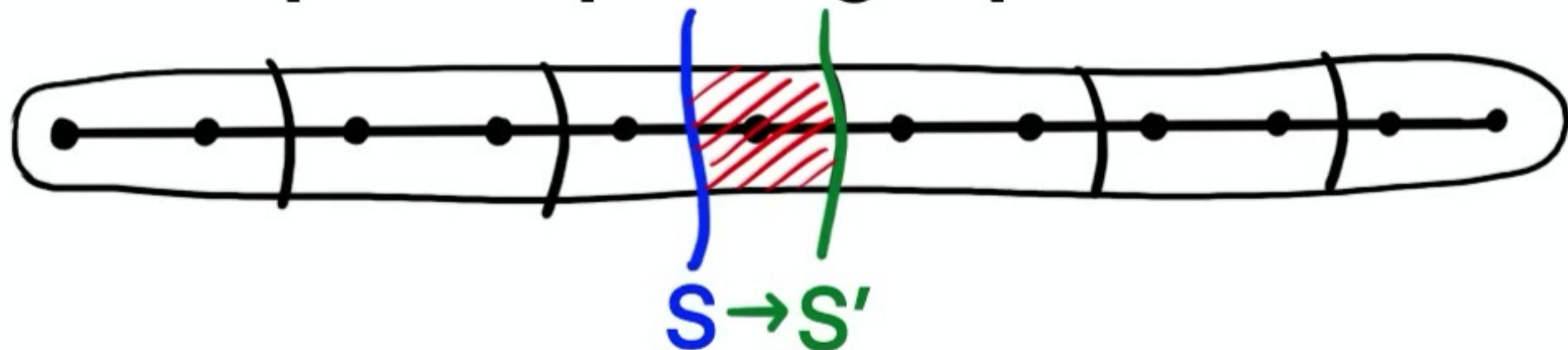
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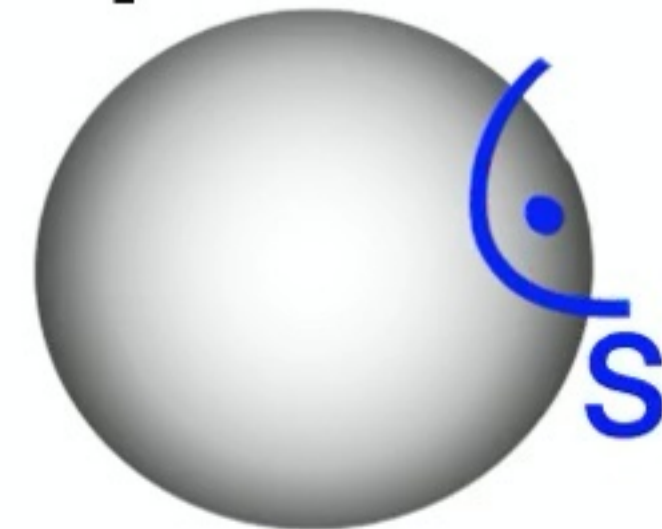
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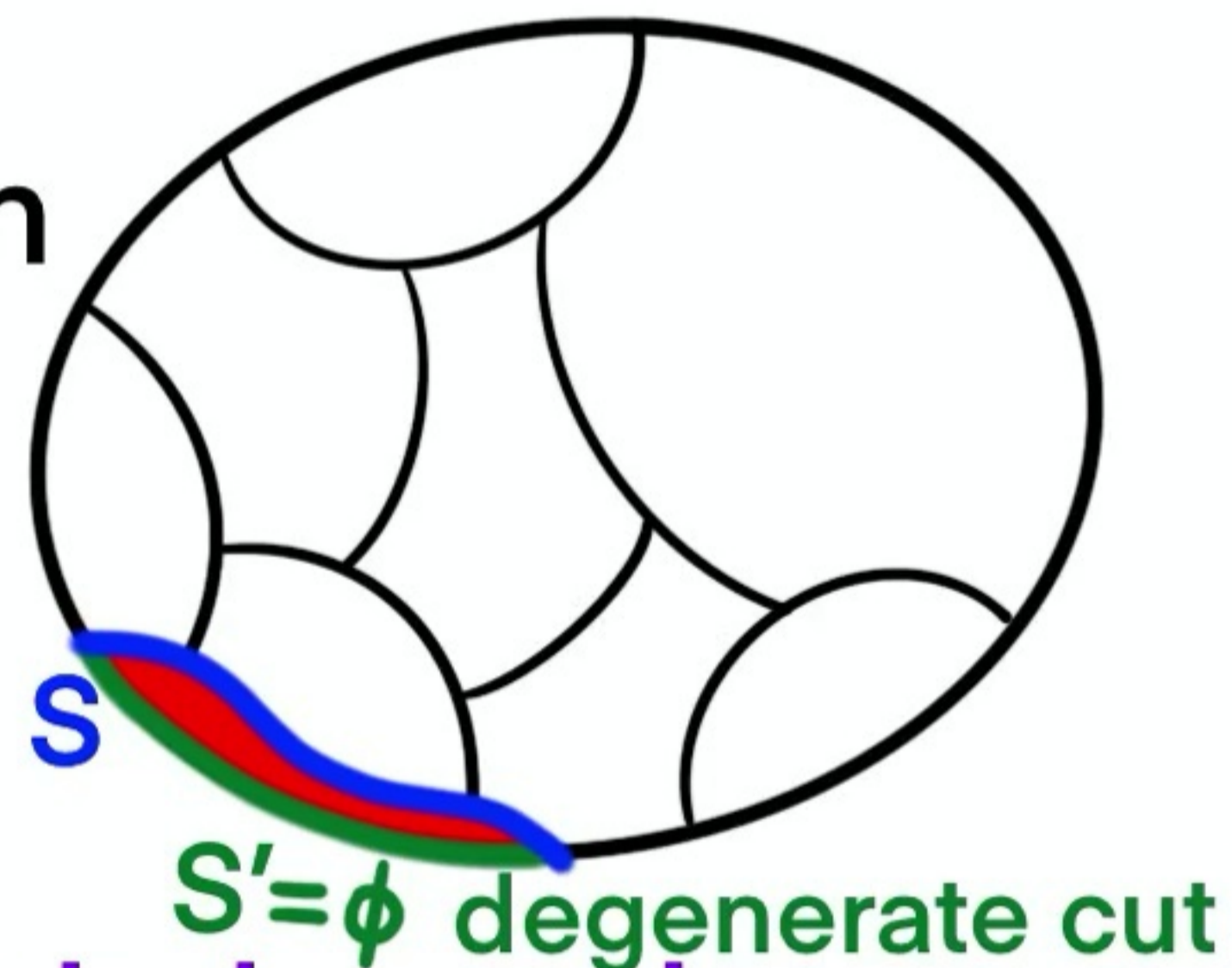
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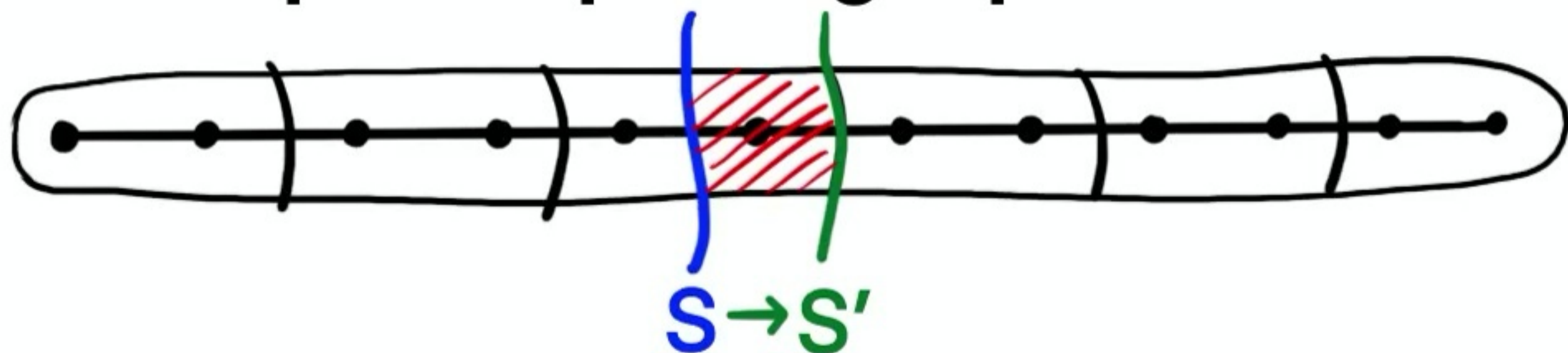
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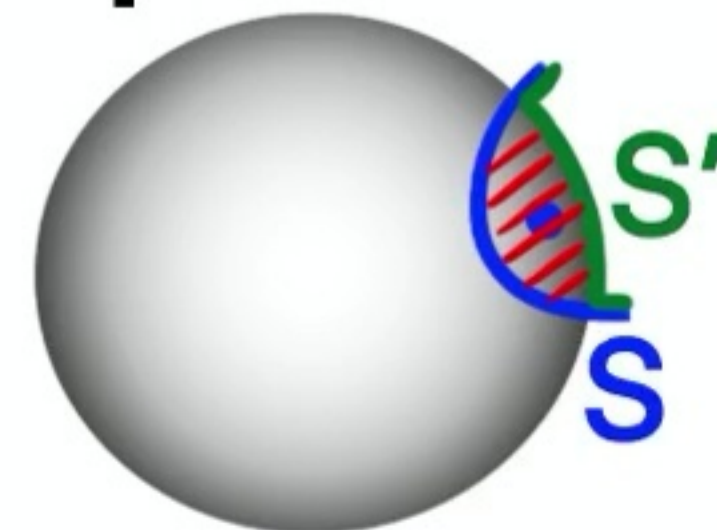
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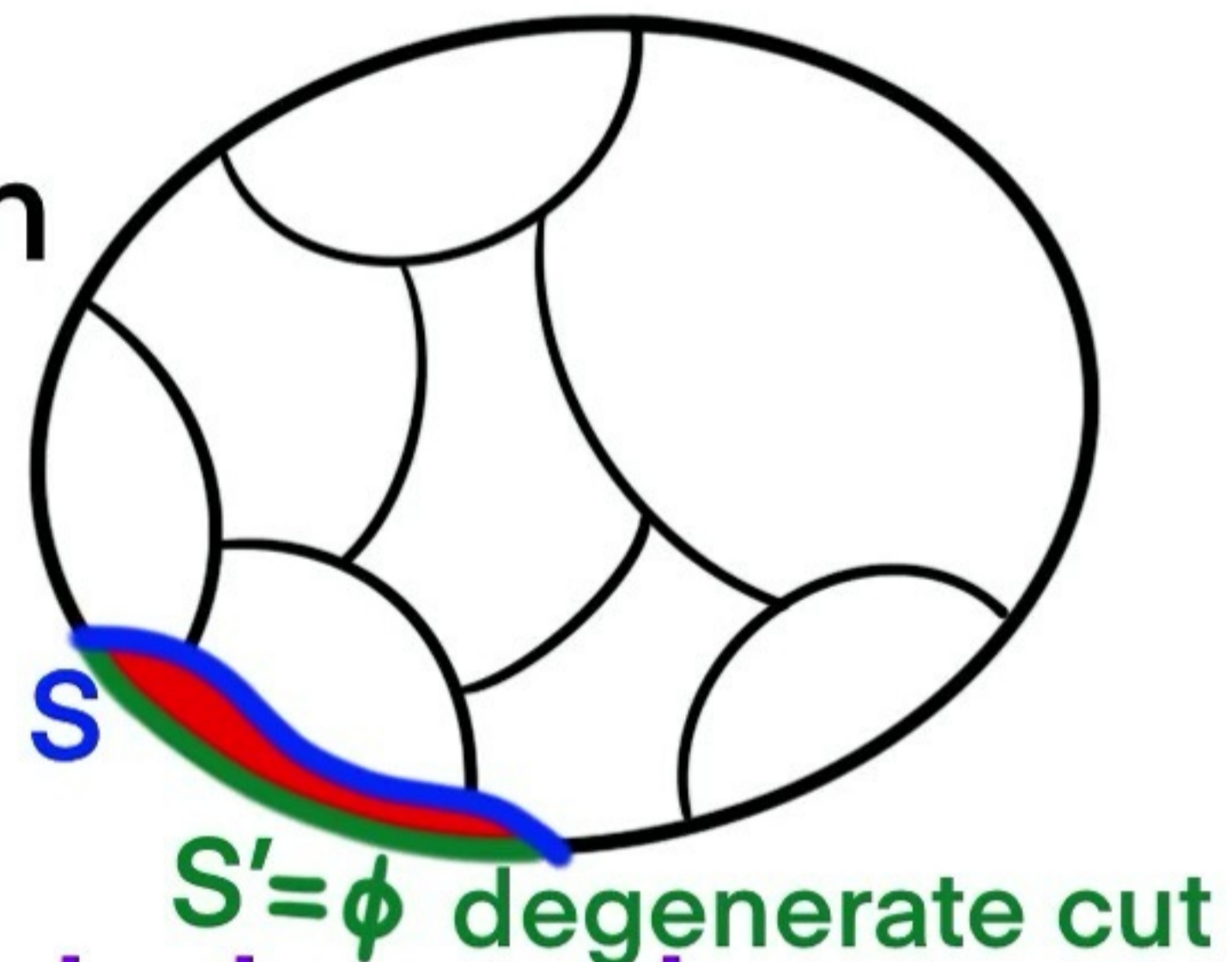
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Algorithm **unbalanced** $\leq \text{poly}(\log n, 1/\epsilon)$ vertices

run local algorithm

- if mincut is **trivial** (~~single vertex on one side~~): ~~find min degree~~

- otherwise, **cluster** and **contract** to preserve global mincut

#vertices $\leq n/2$

up to $(1+\epsilon)$ factor

$(1+\epsilon)$ -approximate mincut

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“Local Karger contraction” [Nalam-Saranurak’23]:

$m k^2 \text{polylog}(n)$ randomized time if k vertices on one side

$k = \text{poly}(\log n, 1/\varepsilon) = \text{polylog}(n)$

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Repeat $\log_2 n$ times: run local algorithm, then cluster+contract
 $\rightarrow (1+\varepsilon)^{\log n}$ approximation. Set $\varepsilon \ll 1/\log n \rightarrow (1+o(1))$ -approx

“Local Karger contraction” [Nalam-Saranurak’23]:

$m k^2 \text{polylog}(n)$ randomized time if k vertices on one side

$k = \text{poly}(\log n, 1/\varepsilon) = \text{polylog}(n)$

Overall: $m \text{polylog}(n)$ randomized for $(1+o(1))$ -approx mincut

Structure Theorem Proof Outline

Theorem: for any weighted undirected graph, can group the vertices into $\leq n/2$ clusters s.t.

- for any 1.01-approx mincut S , can modify few vertices $\rightarrow S'$ s.t.
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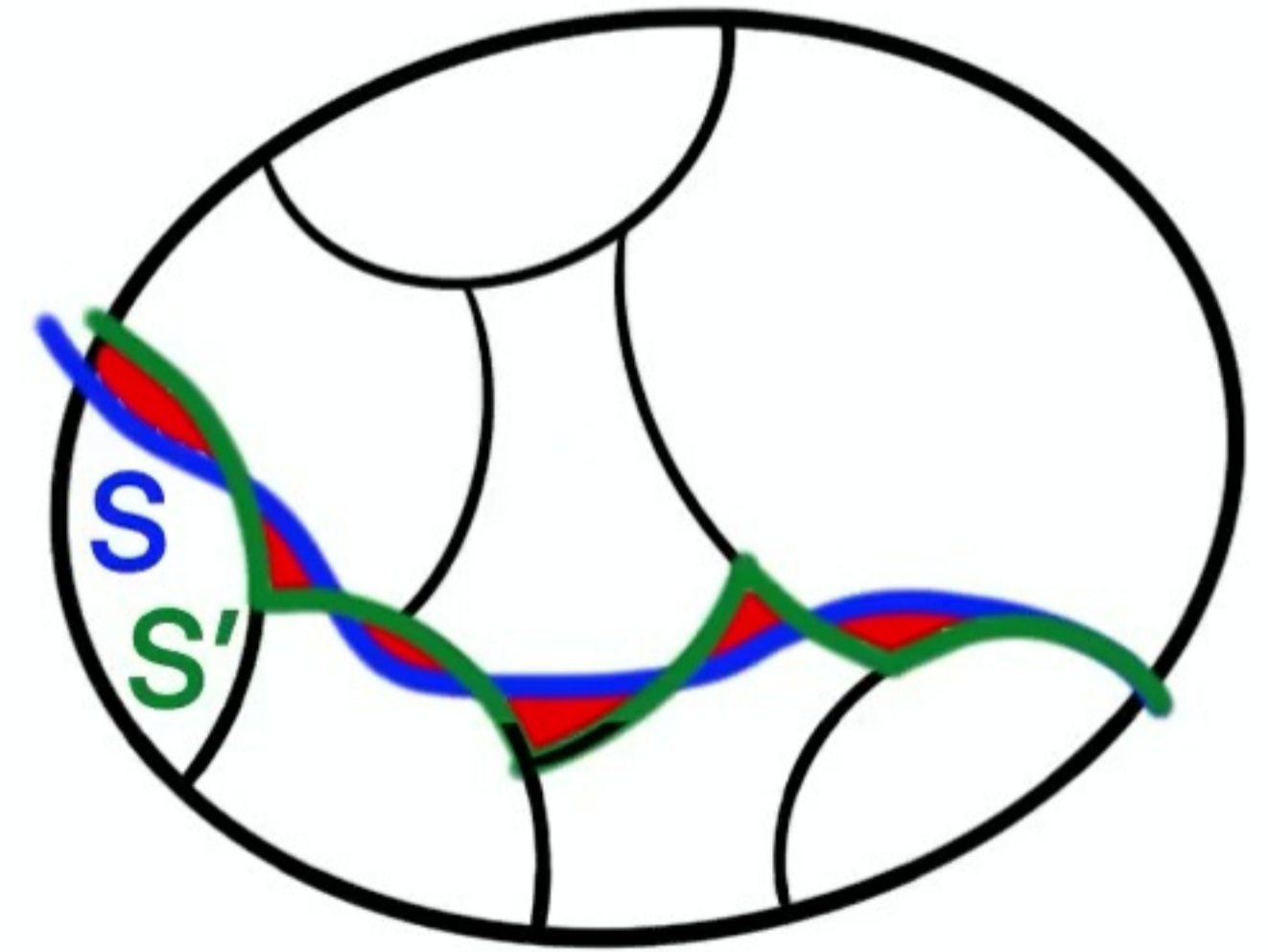
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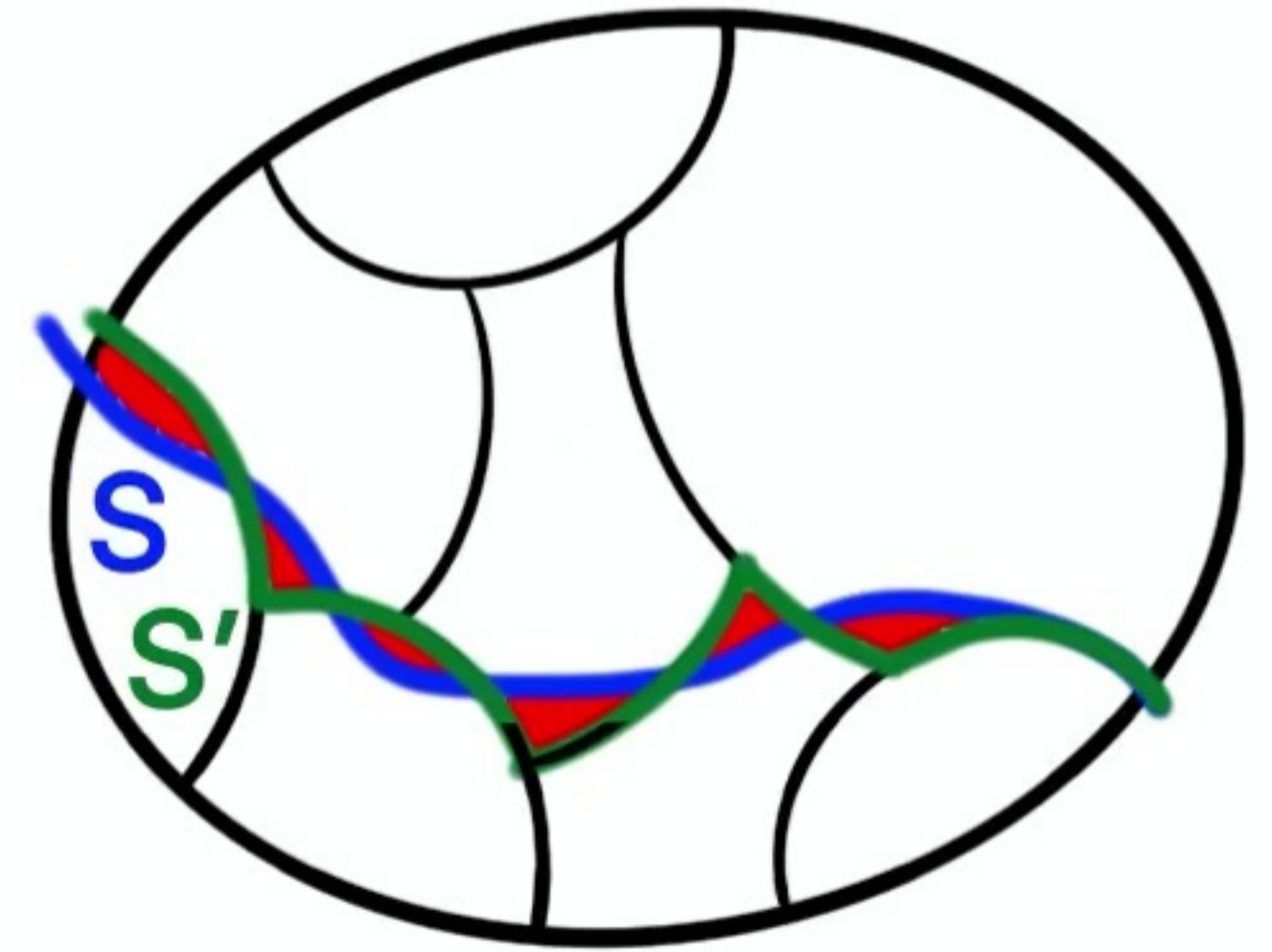


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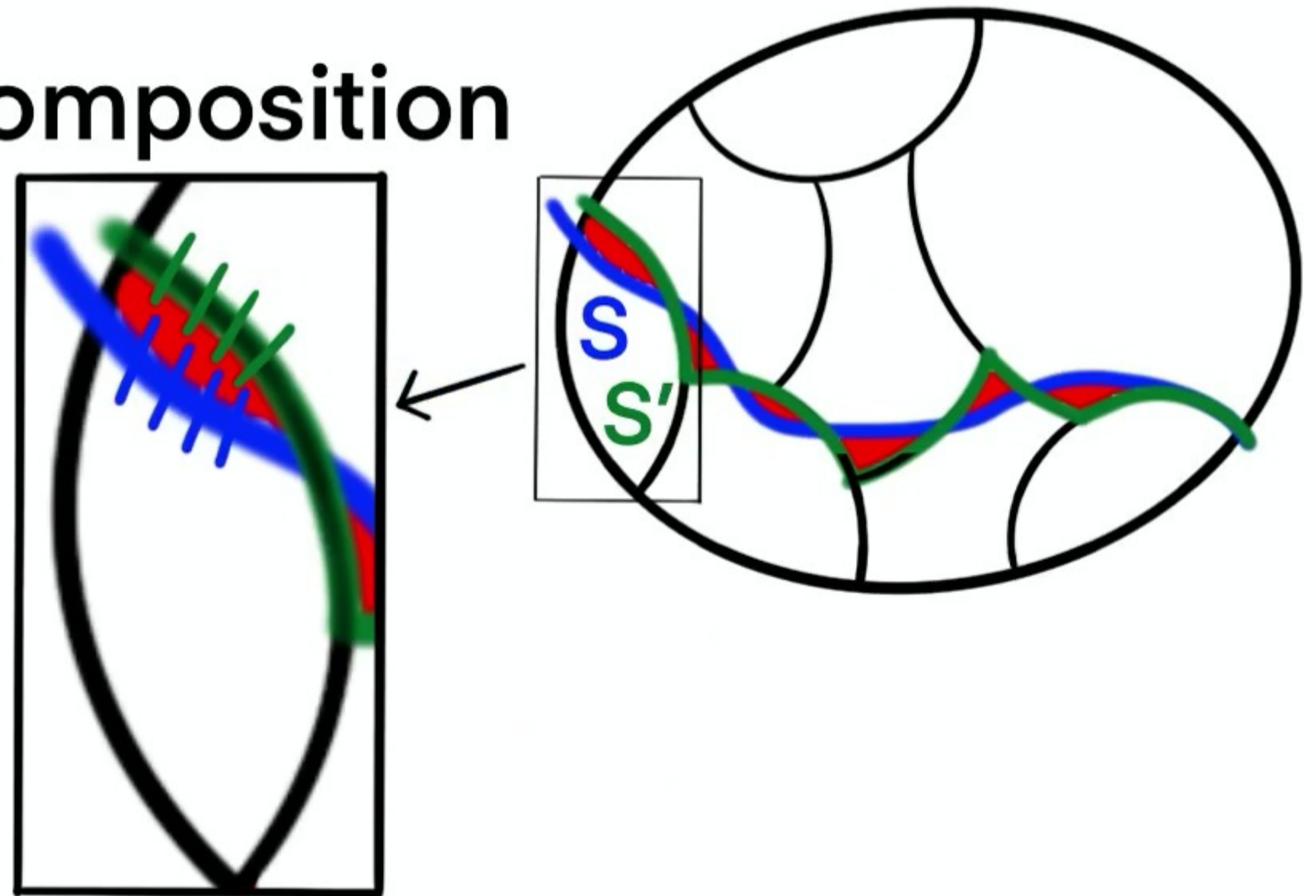


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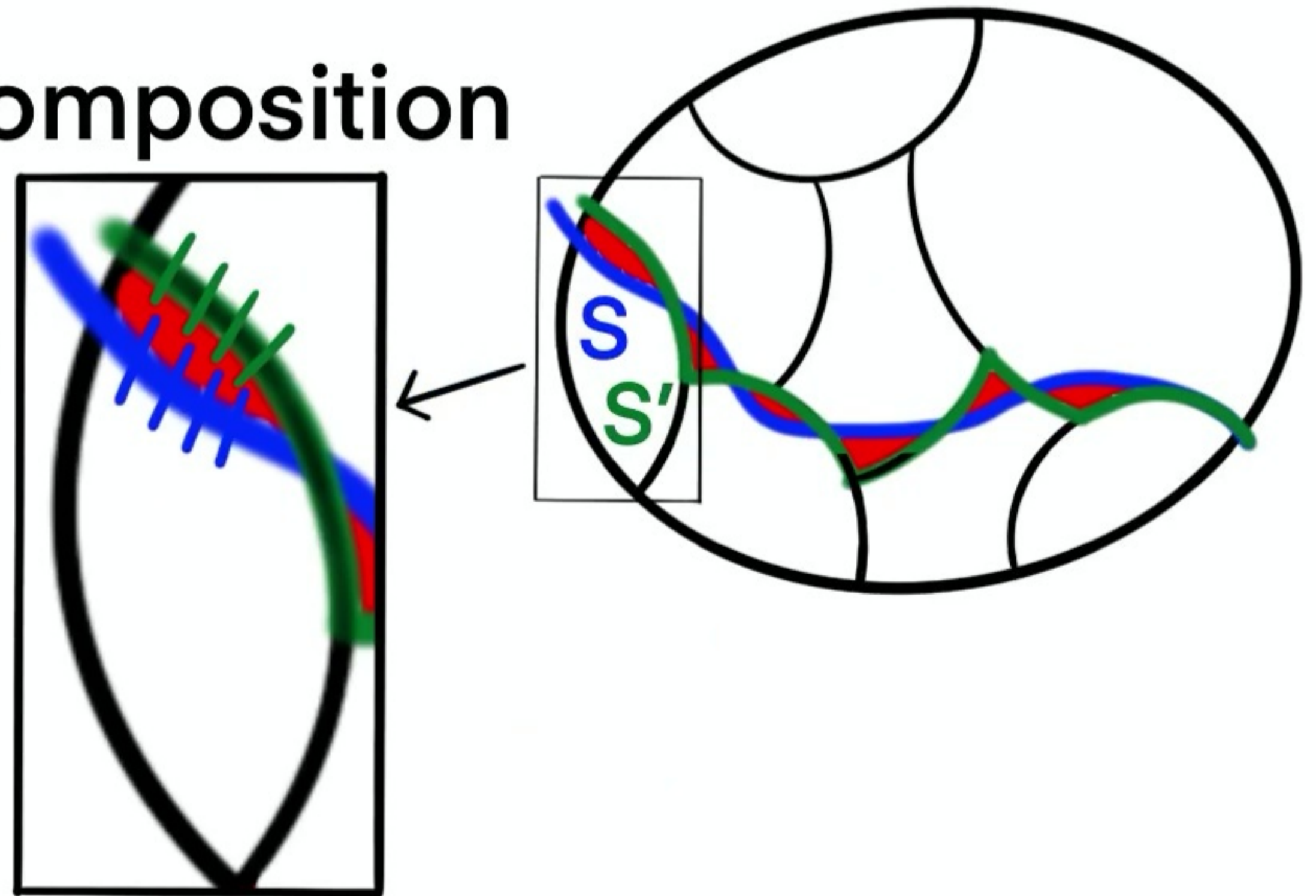
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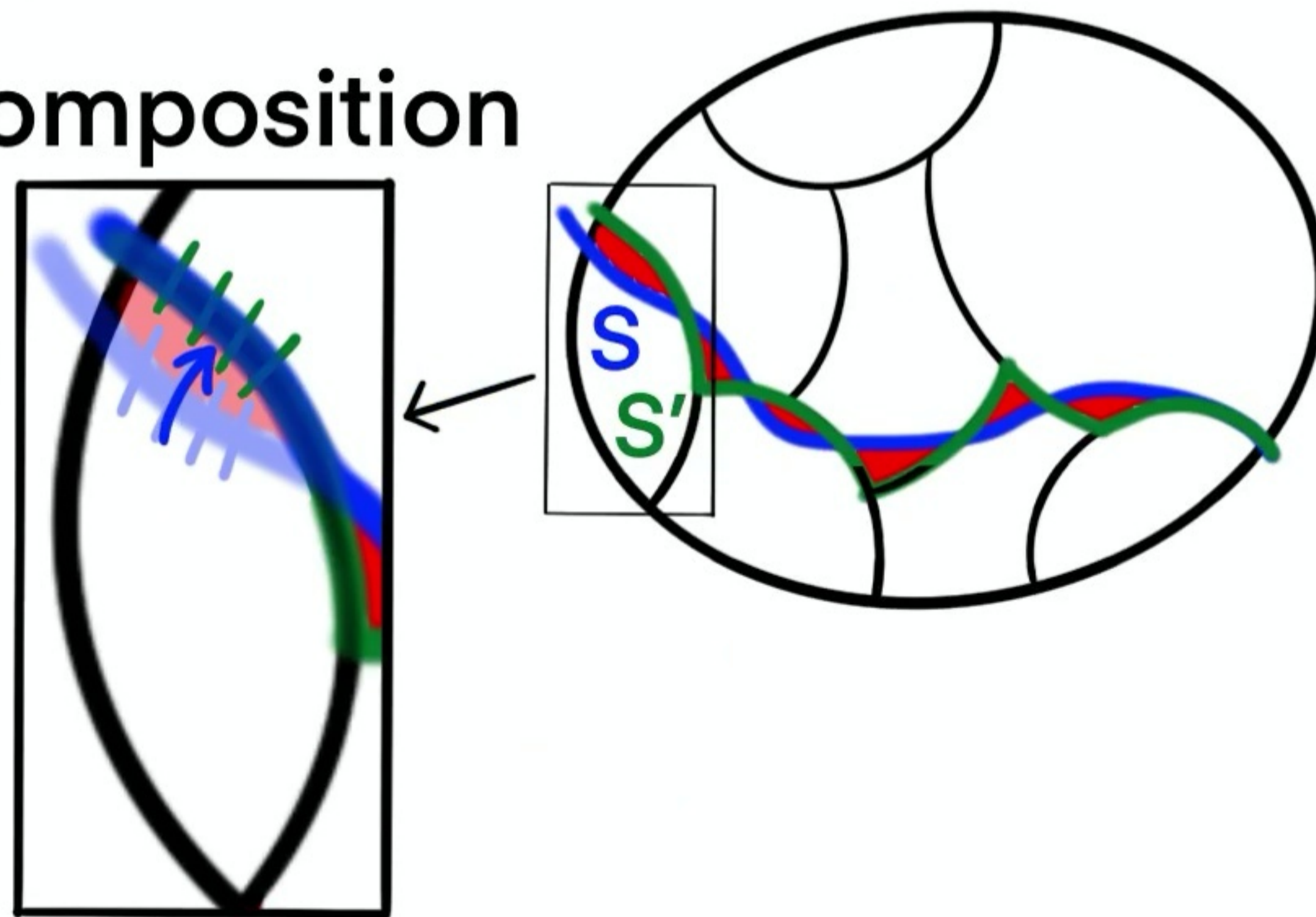
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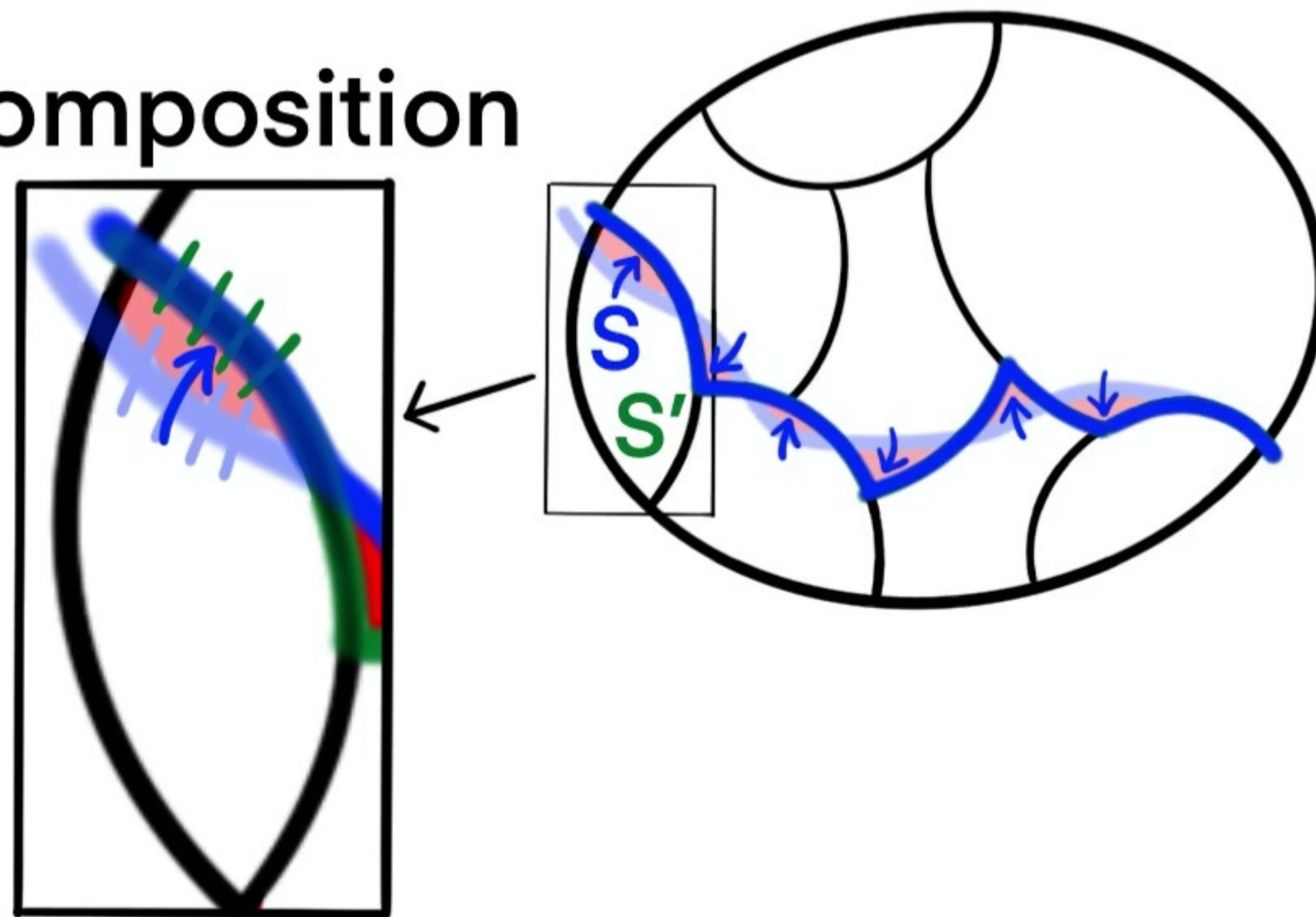
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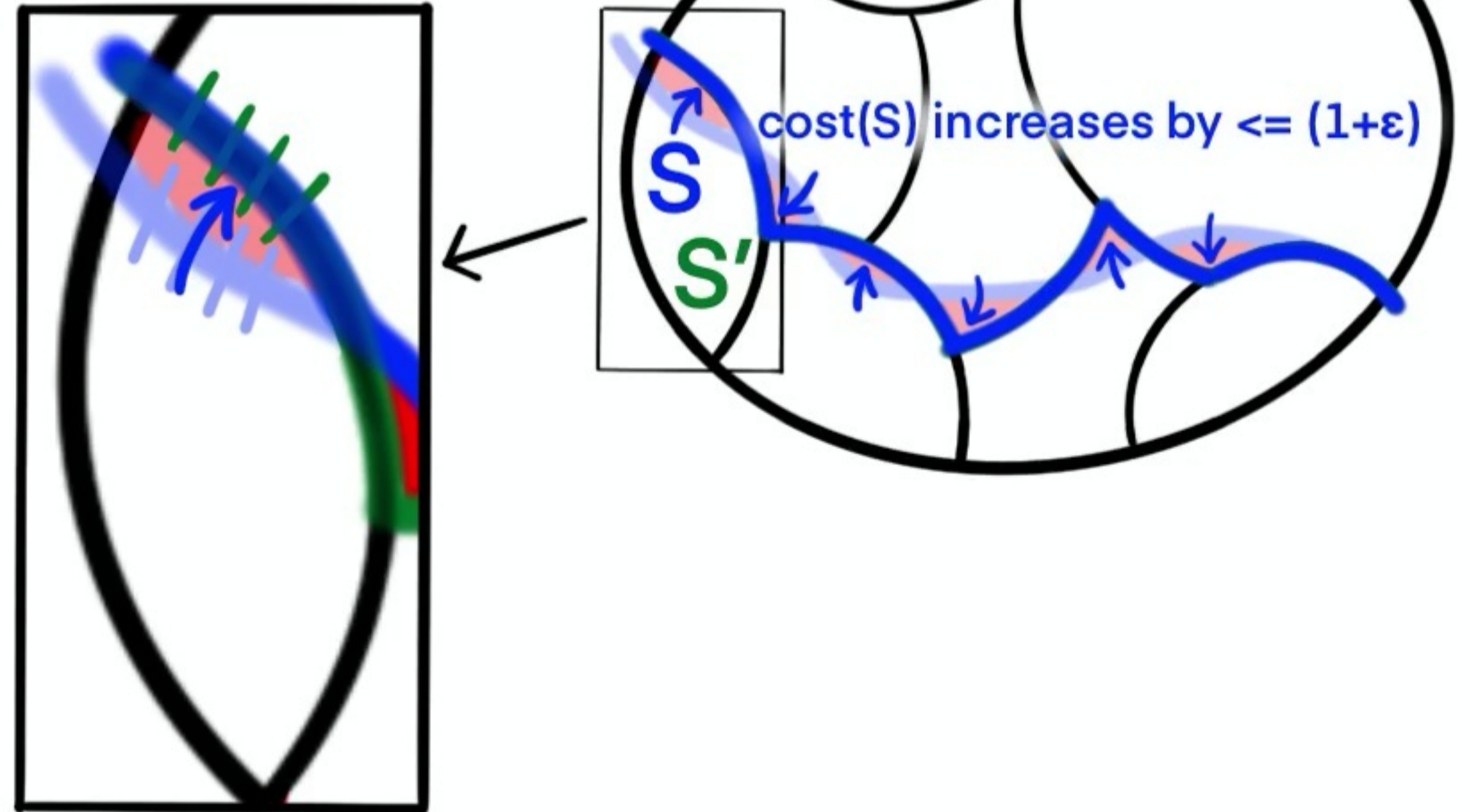
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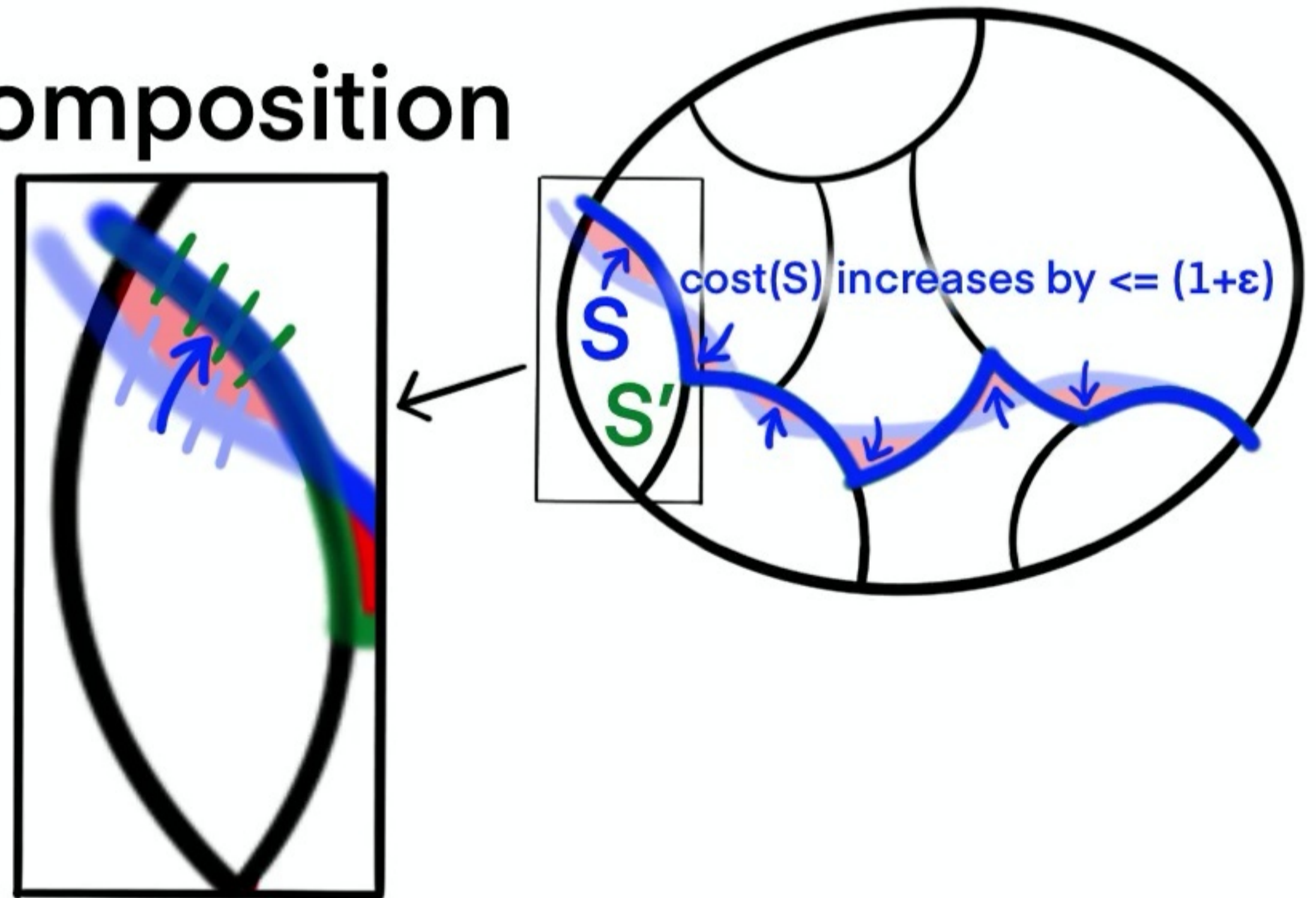
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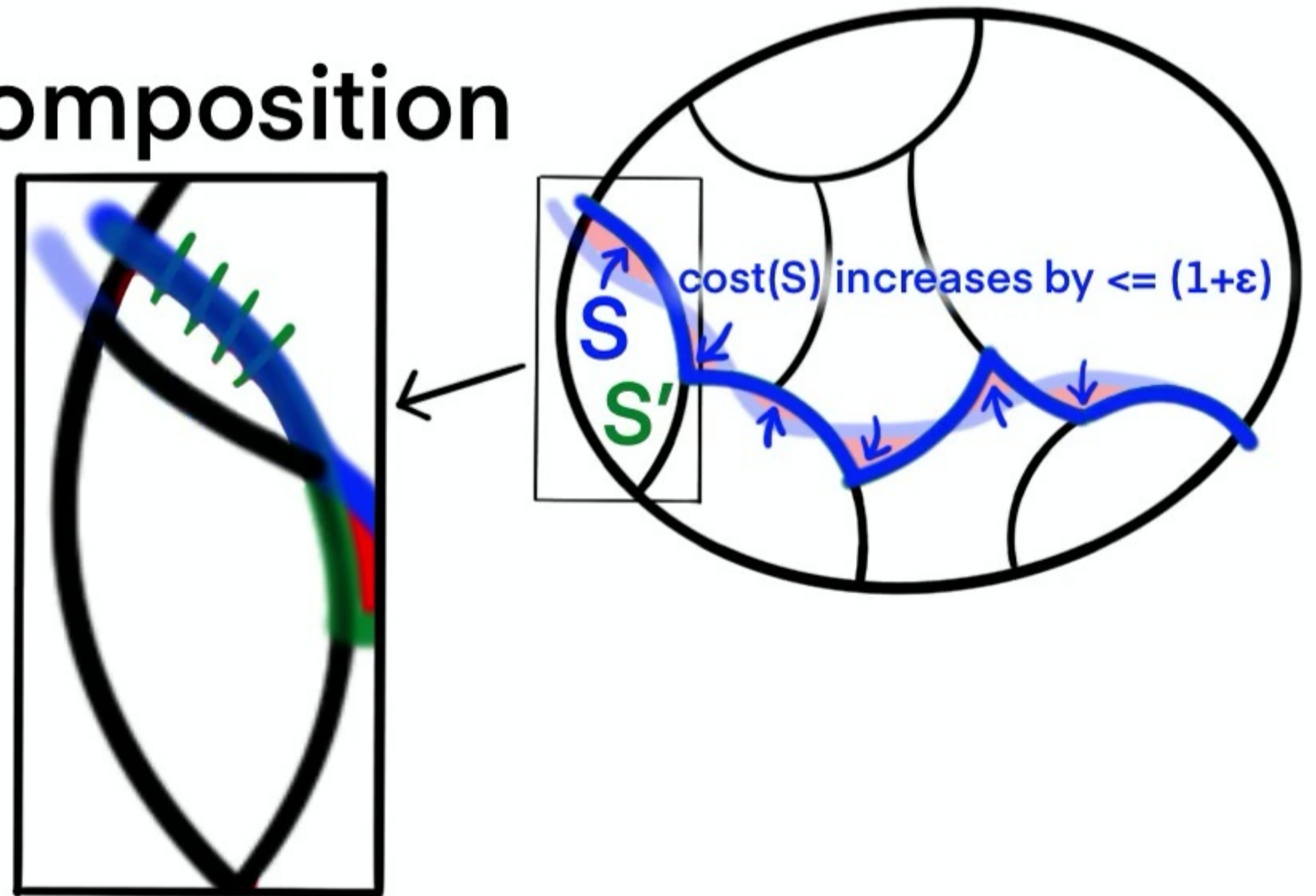
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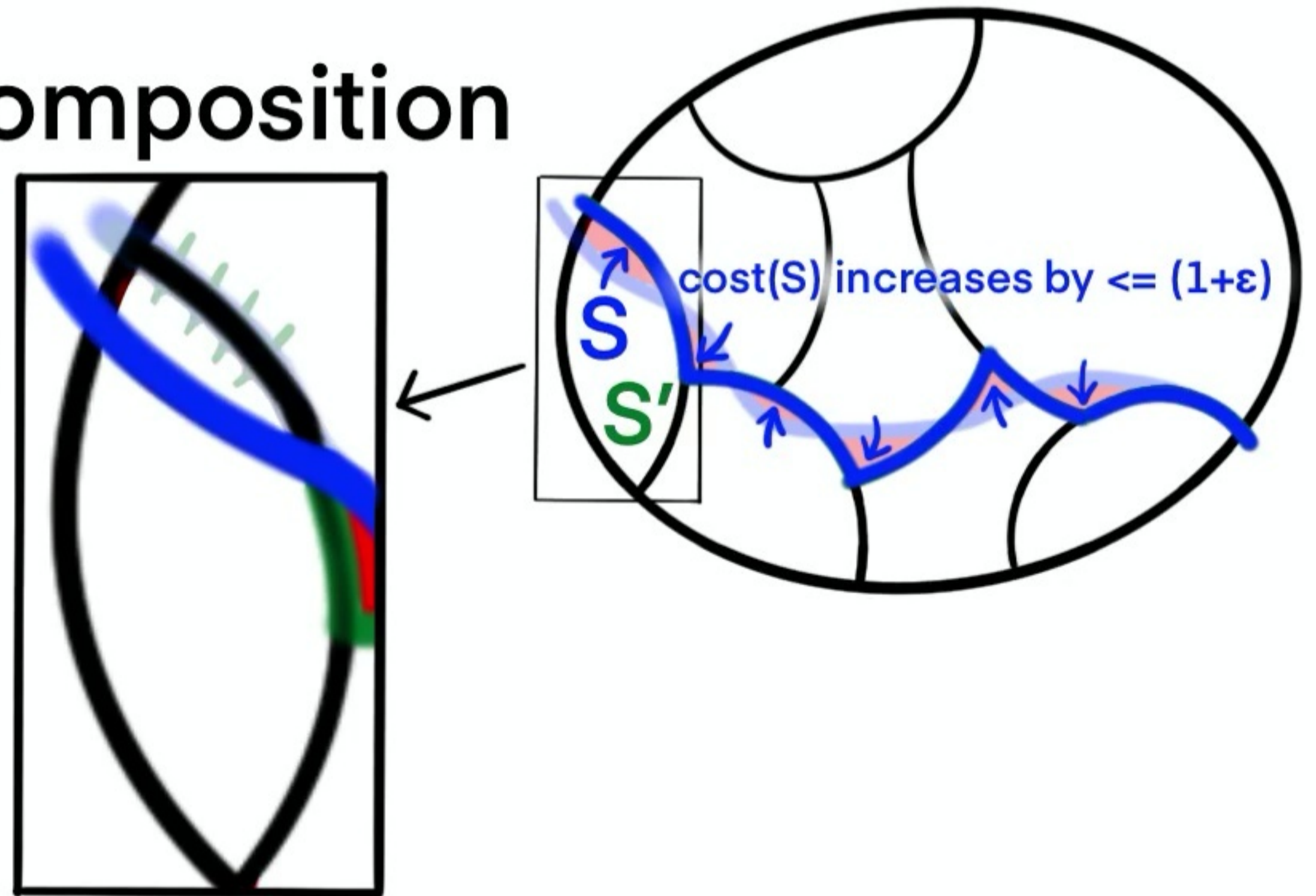
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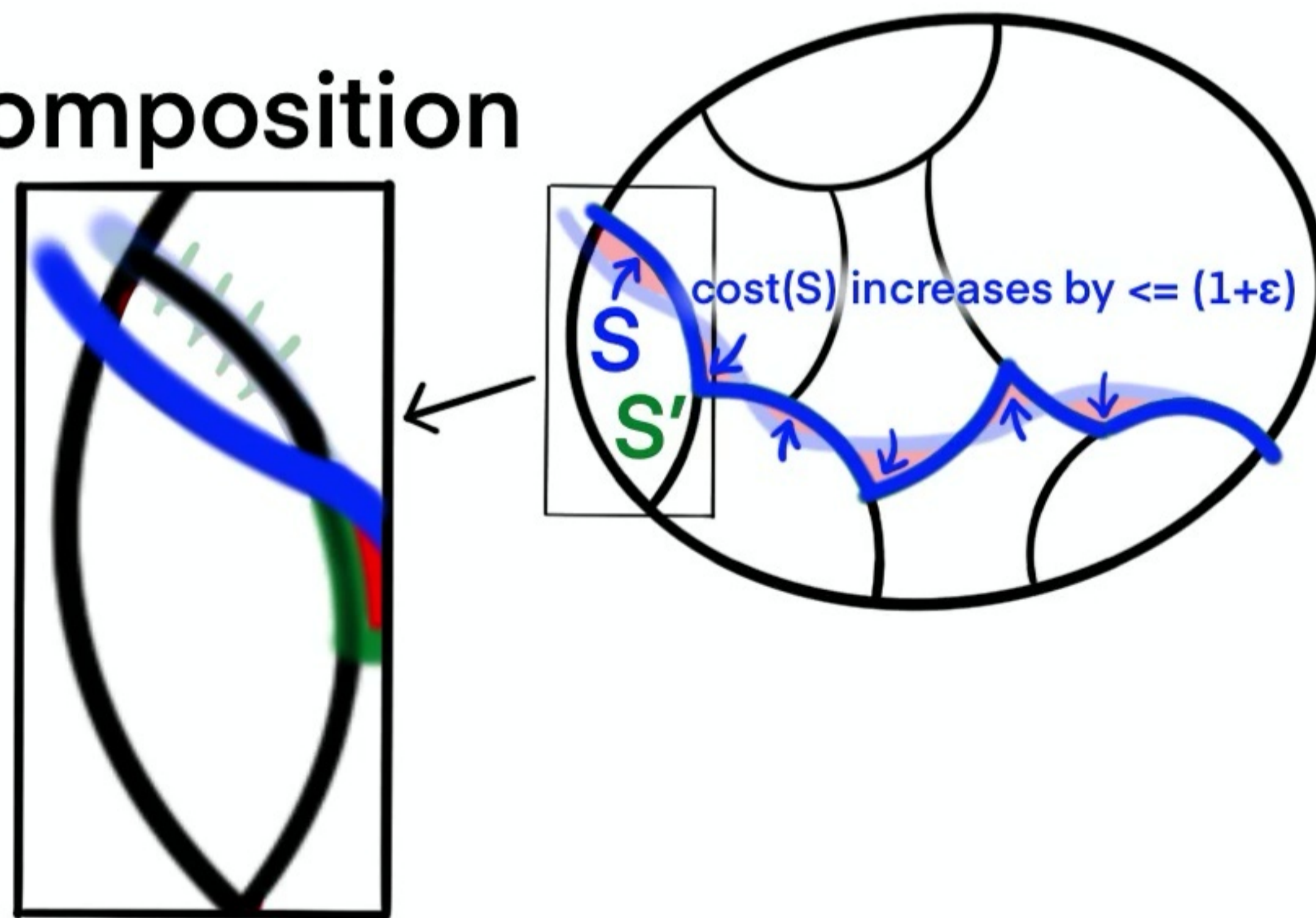
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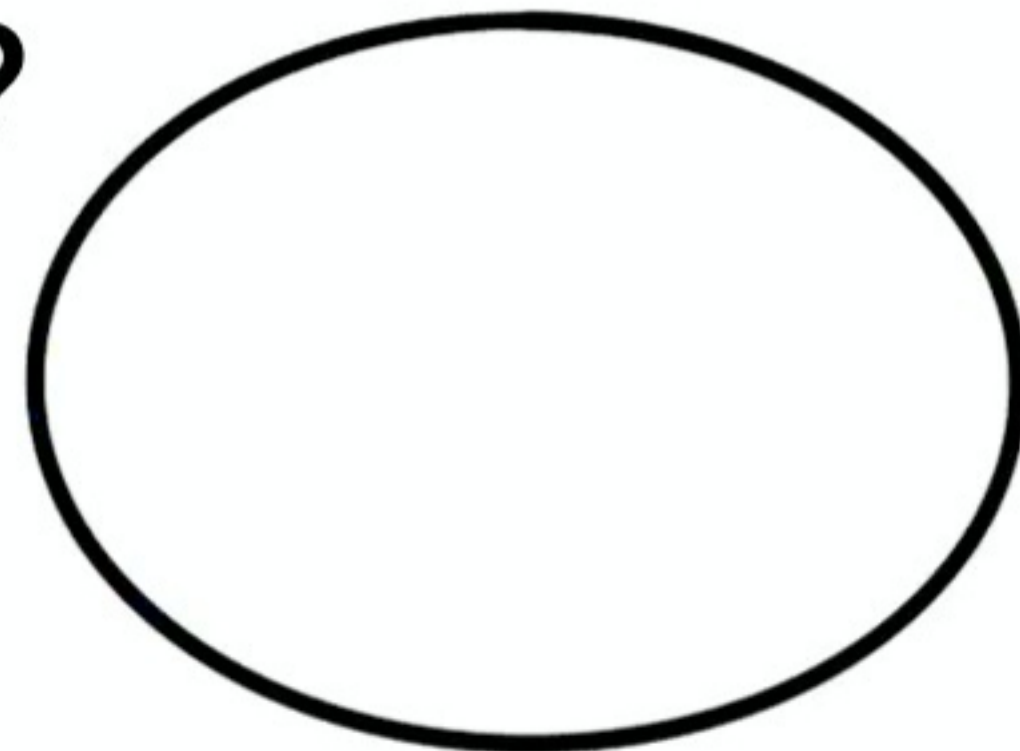
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- **Should be useful for global mincut in other settings (dynamic/streaming/distributed)**