Faster minimum k-cut of a simple graph

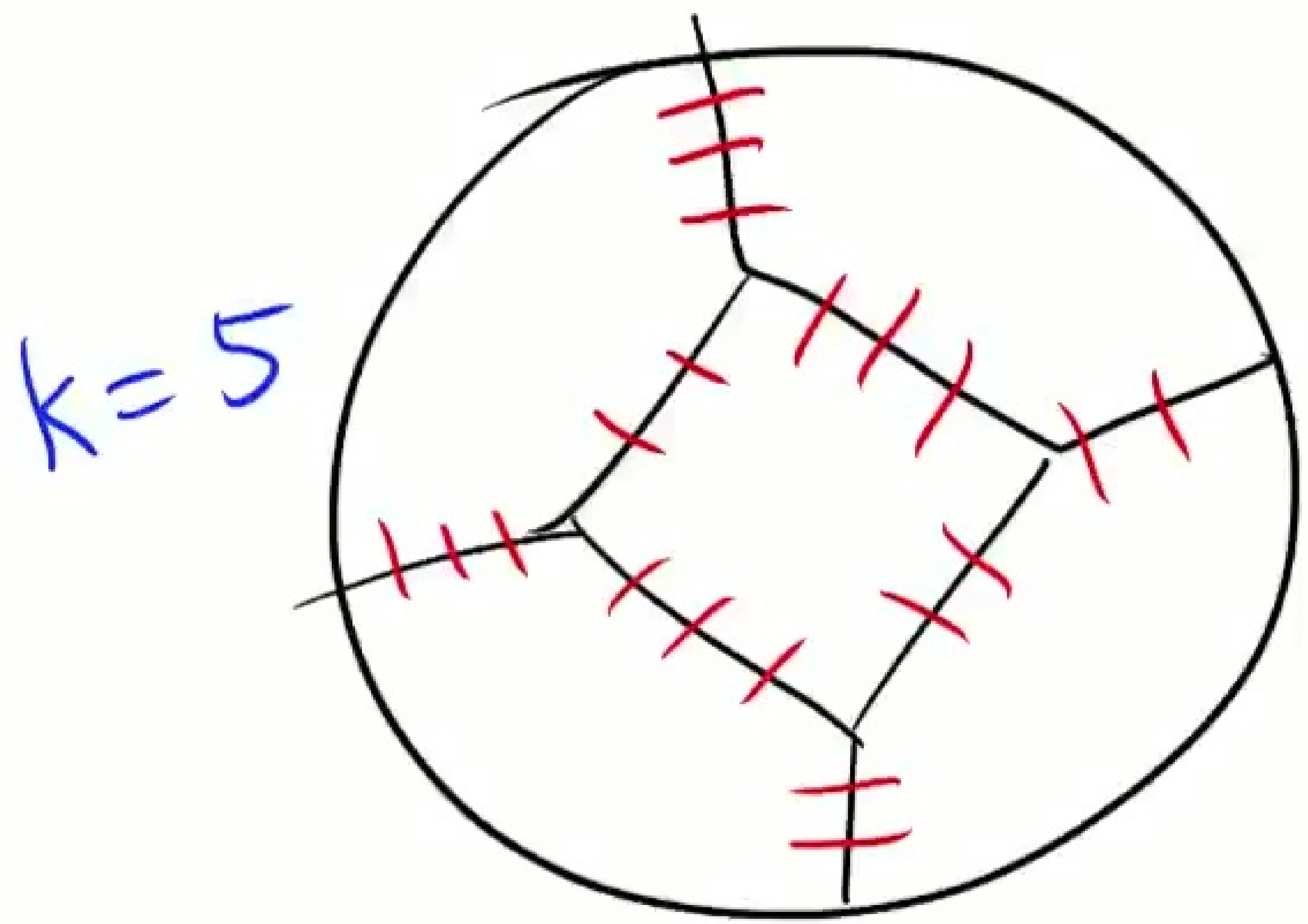
Jason Li

STOC 2019
November 11, 2019

Introduction

minimum k-cut: delete min weight edges to cut graph into $\geq k$ connected components

Setting: exact algorithm, k constant



Prior Work

Goldschmidt–Hochbaum 1994: $O(n^{(1/2-o(1))k^2})$ time deterministic

Karger–Stein 1994: $\tilde{O}(n^{2(k-1)})$ time randomized

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Same authors, 2018:	$O_k(n^{(2\omega/3+o(1))k})$ time deterministic, integer weights $\leq n^{O(1)}$
This work:	$O_k(n^{(1+o(1))k})$ time randomized for simple graphs (unweighted, no multi-edges)

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Lower bound: as hard as k-clique: $\Omega(n^{(\omega/3-o(1))k})$ time algebraic

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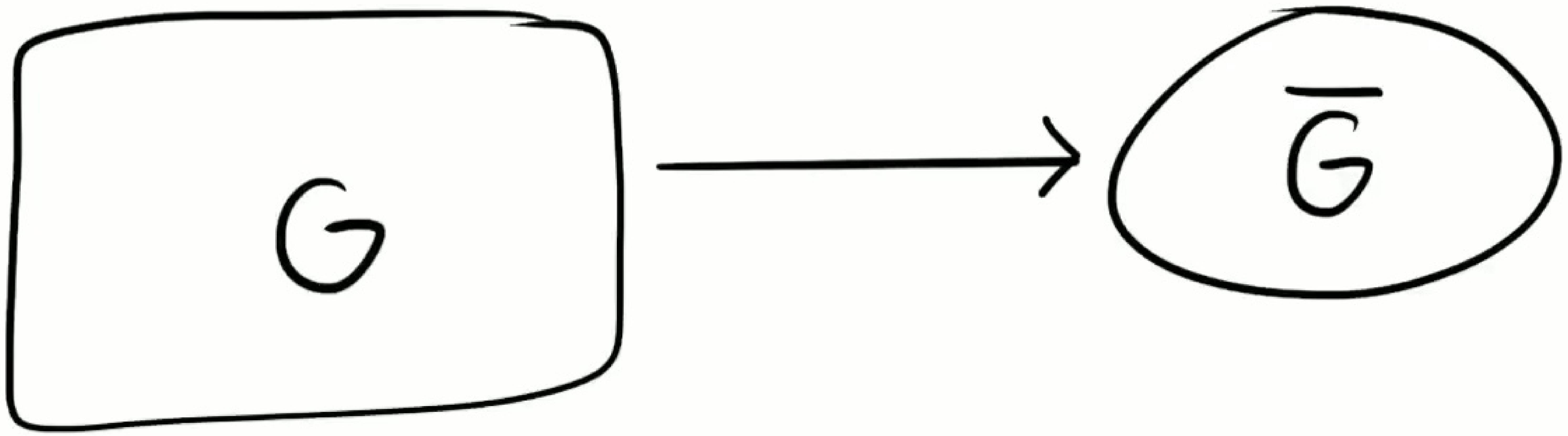

min k-cut $= \lambda_k$

Kawarabayashi-Thorup Sparsification

- $\bar{n} := n/\lambda_k$ vertices
- Preserves min k-cut

  (if it's nontrivial)

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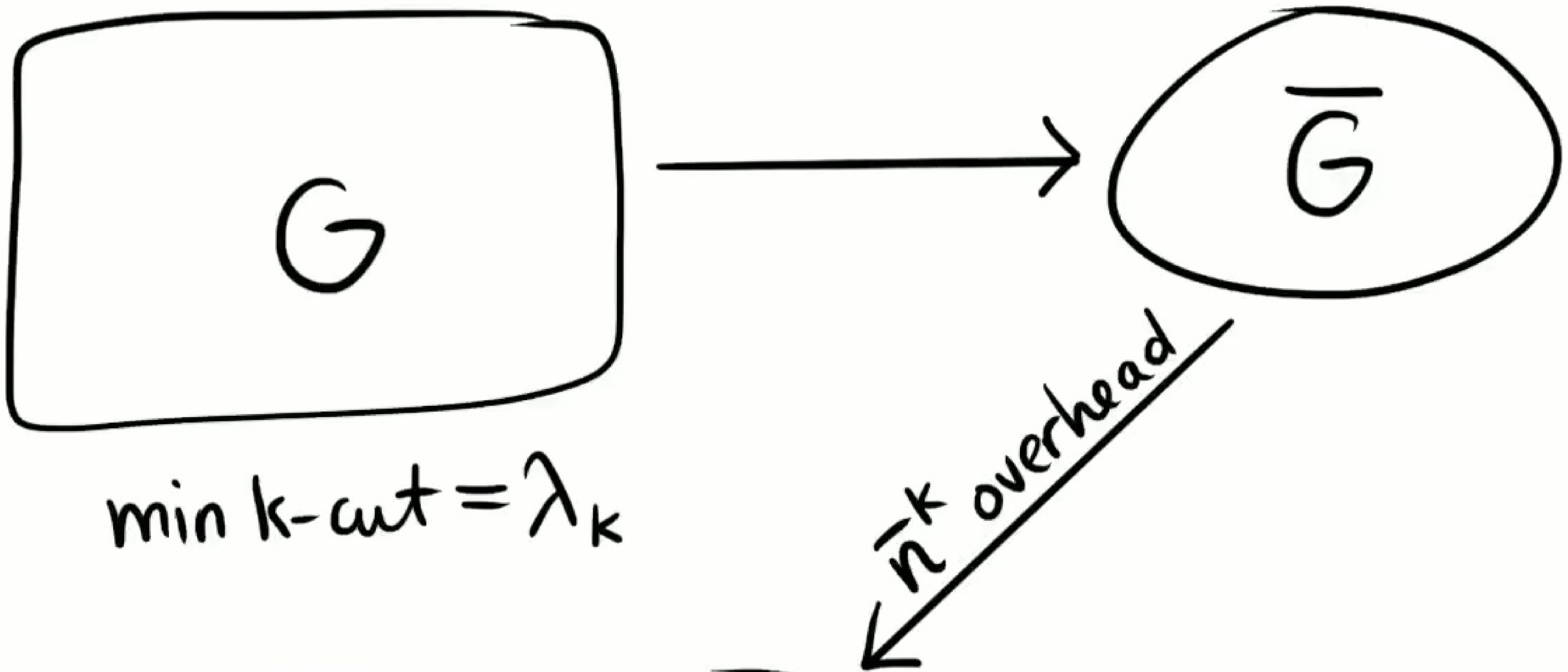

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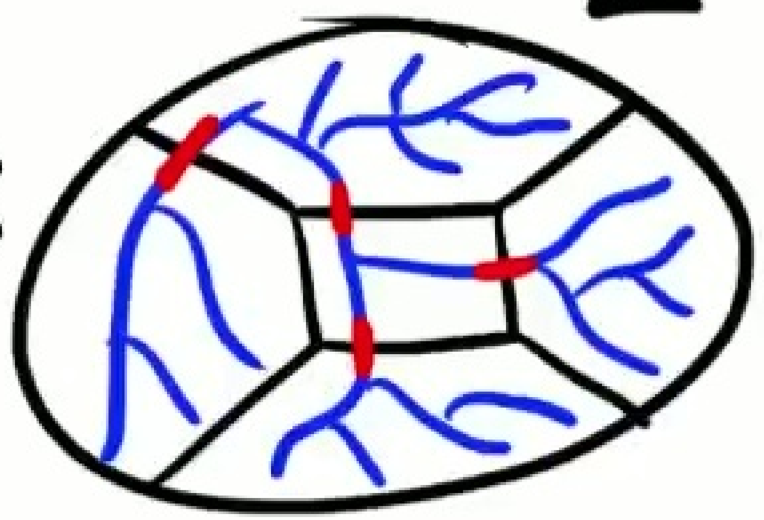

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$\bar{n}^k$ trees s.t.
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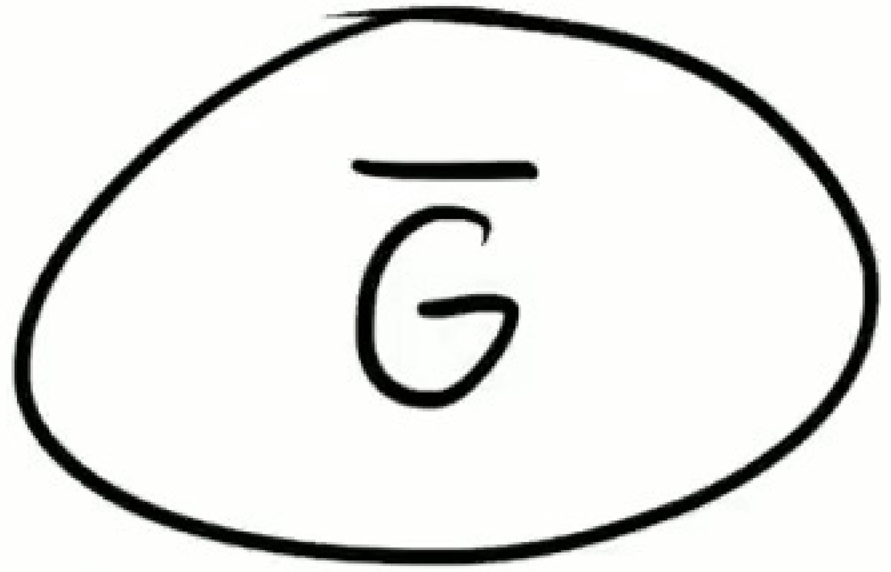

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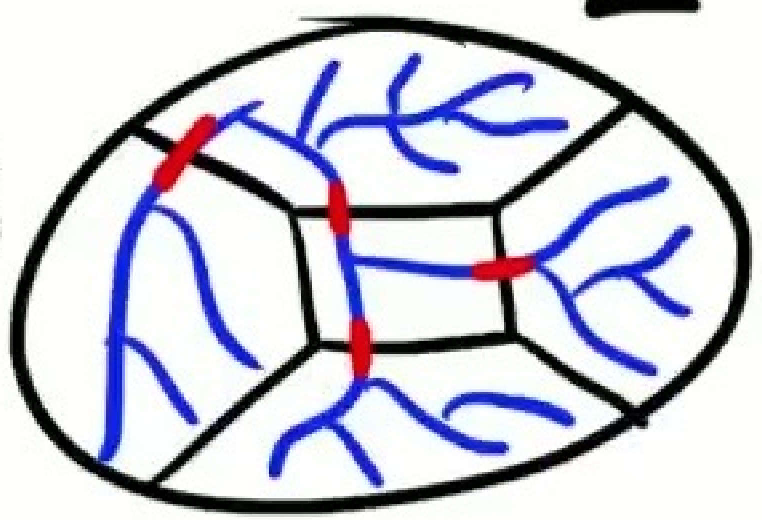

Solve tree problem: cut best k-1 edges

- color-coding (this talk)
- heavy-light decomposition
- $\lambda_k^k \bar{n}^{o(k)}$ time

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$G \longrightarrow \bar{G}$

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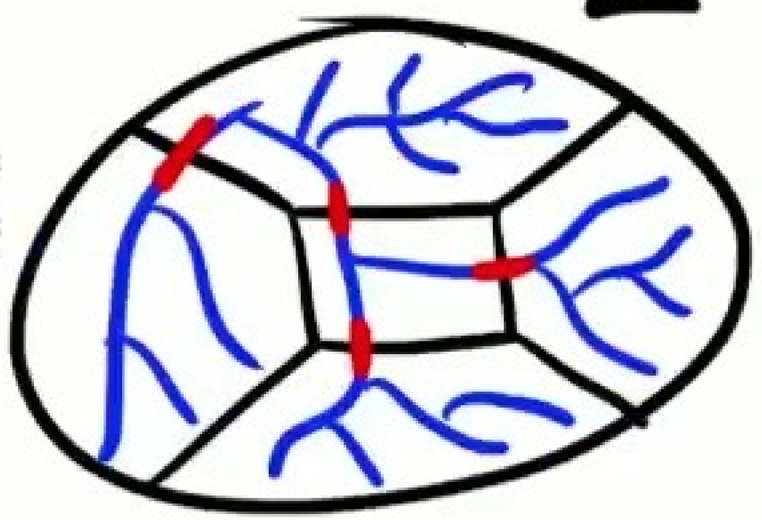

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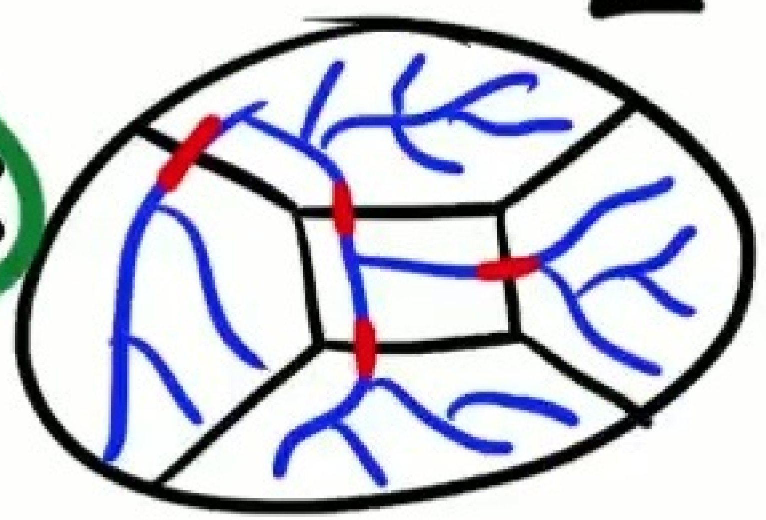

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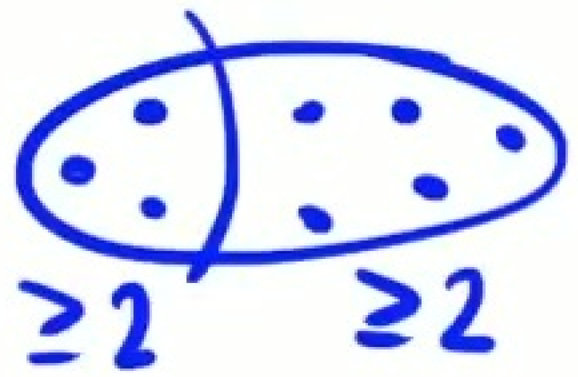
    at least two vertices on each side 

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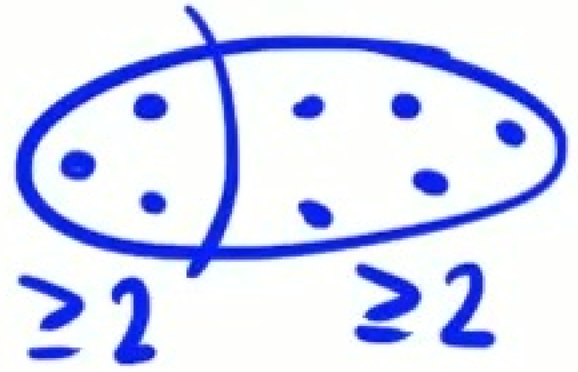
    at least two vertices on each side 

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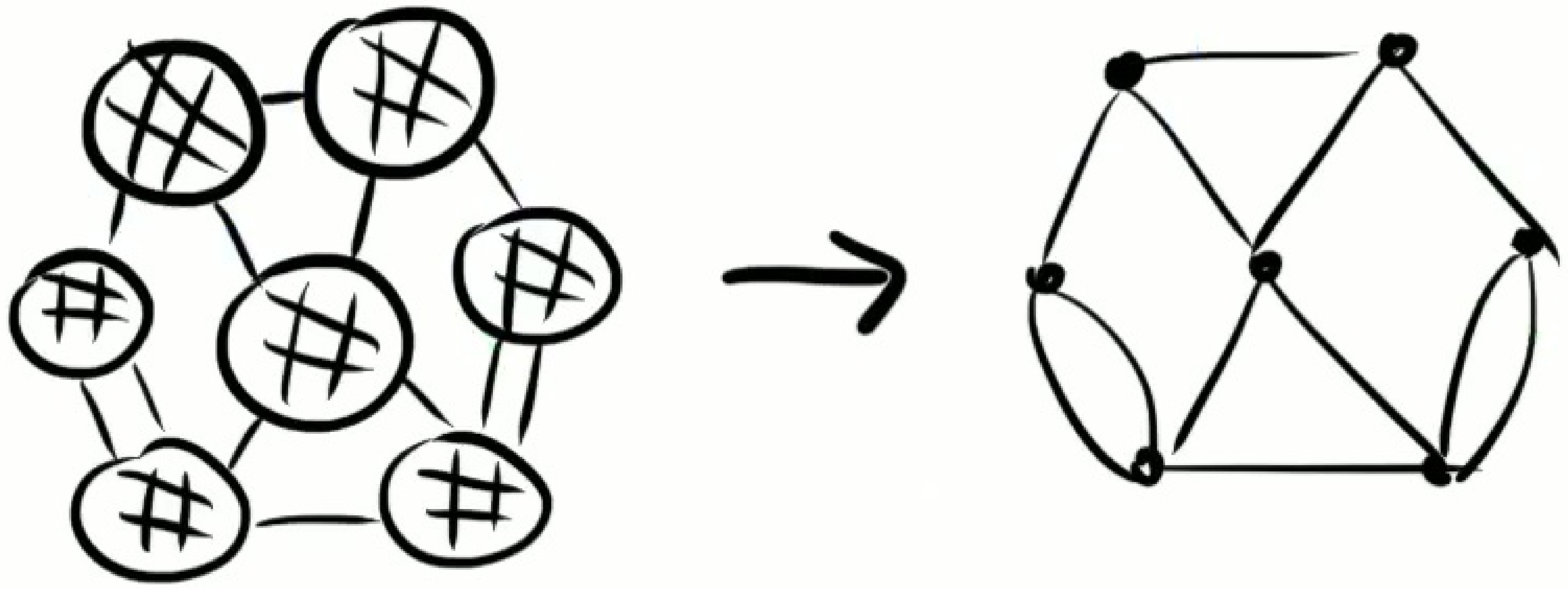

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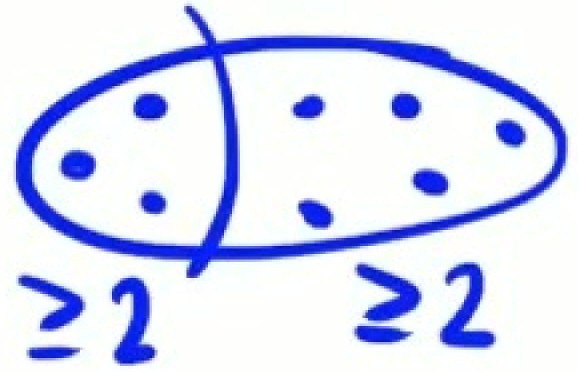

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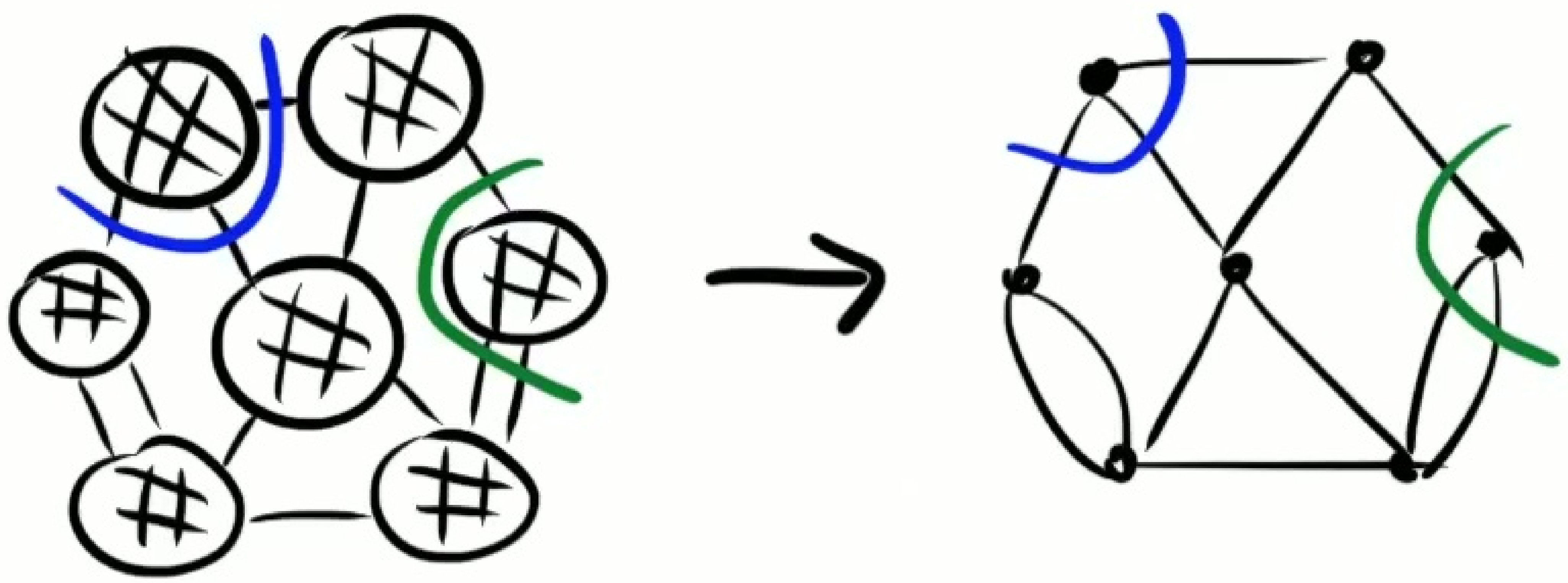

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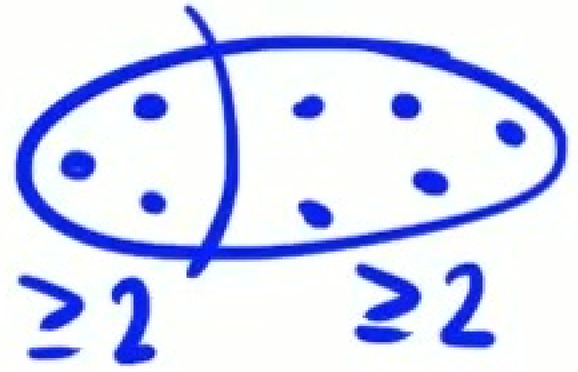

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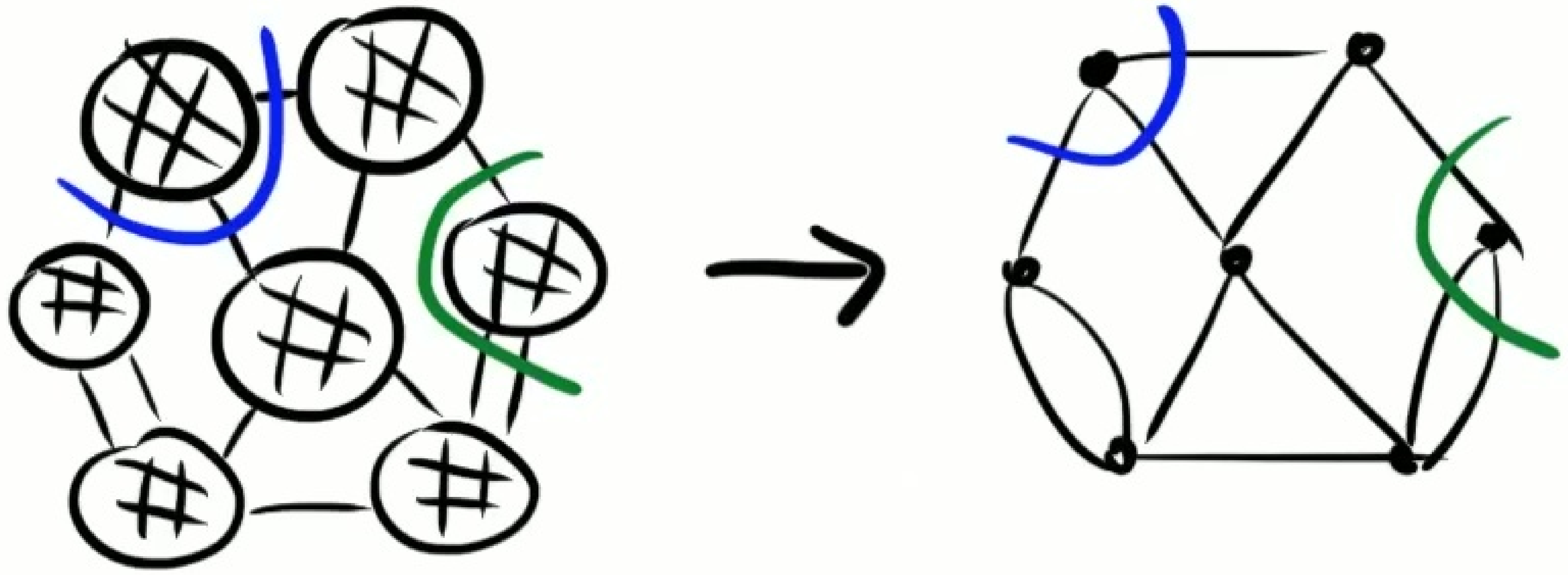

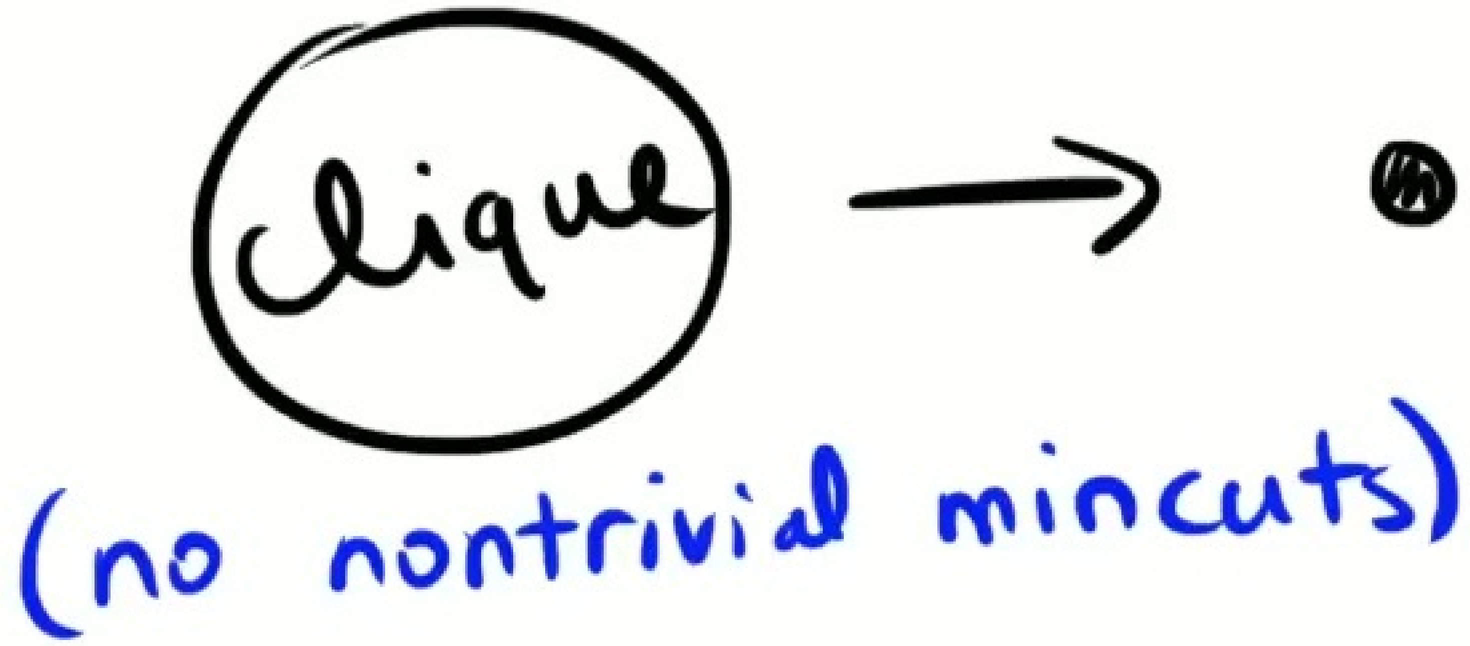

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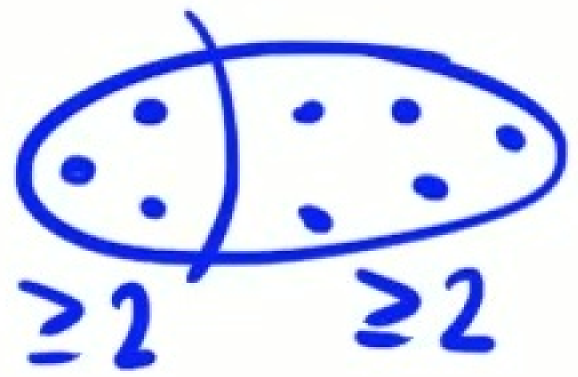
    at least two vertices on each side 

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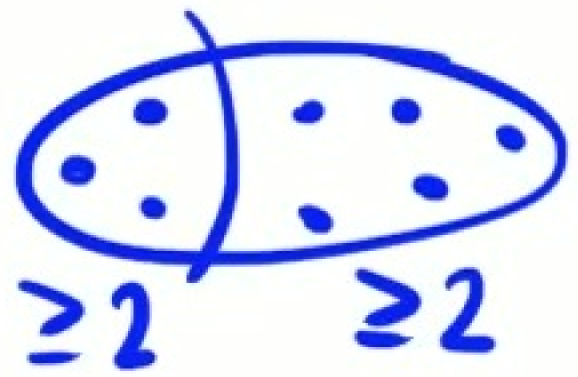

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  Trivial mincuts easy

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For any simple graph G with (min k-cut) $=: \lambda_k \le f(k)\cdot\lambda$, can contract edges into multigraph $\bar{G}$ s.t.
(regularity)

- no nontrivial ~~mincut~~ min k-cut has any edges contracted

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- number of vertices is $\tilde{O}_k(n/\lambda_k)$
- Used for ~~deterministic~~ ~~mincut~~ min k-cut: sparsify into $\cancel{\bar{m}=\tilde{O}(m/\lambda)}$ $\bar{n}=\tilde{O}(n/\lambda_k)$

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  Trivial min k-cuts easy (guess one vertex and recurse)

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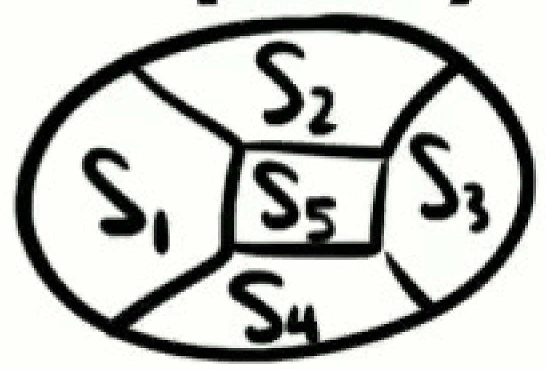
Given weighted graph, exists collection $\mathcal{T}$ of poly(n) spanning trees of G s.t. for any min k-cut $S_1, S_2, \ldots, S_k \subseteq V$ exists tree $T \in \mathcal{T}$ s.t. $|E_T[S_1, S_2, \ldots, S_k]| \leq 2k-2$



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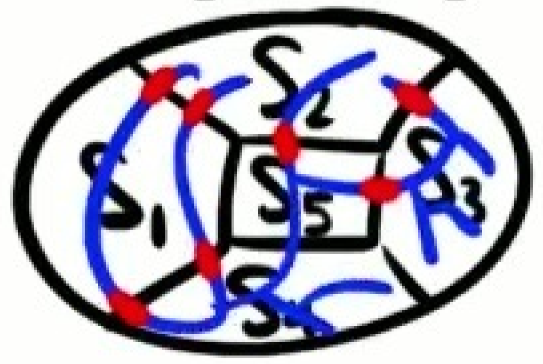

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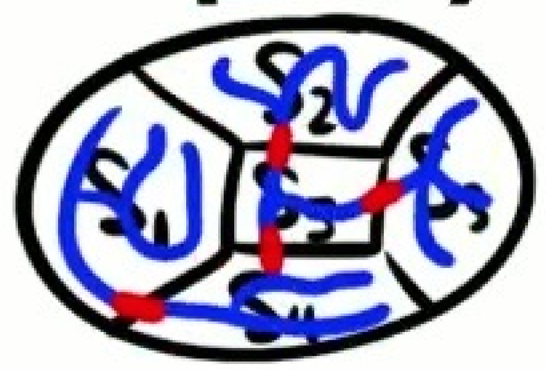

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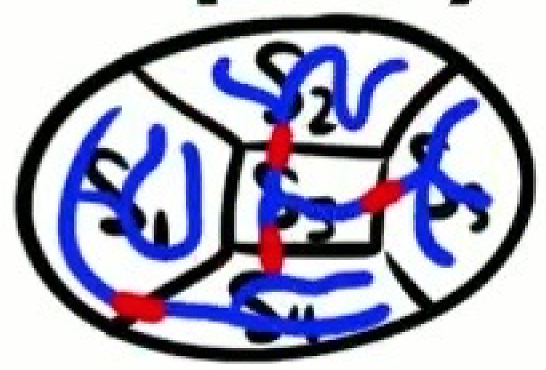

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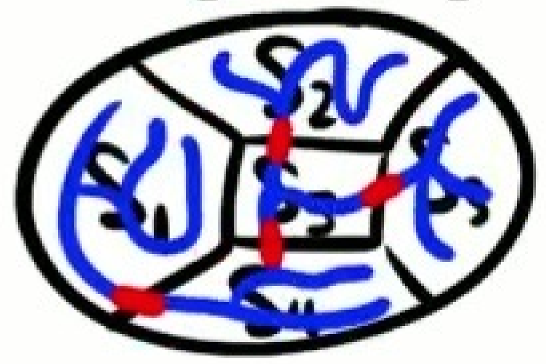

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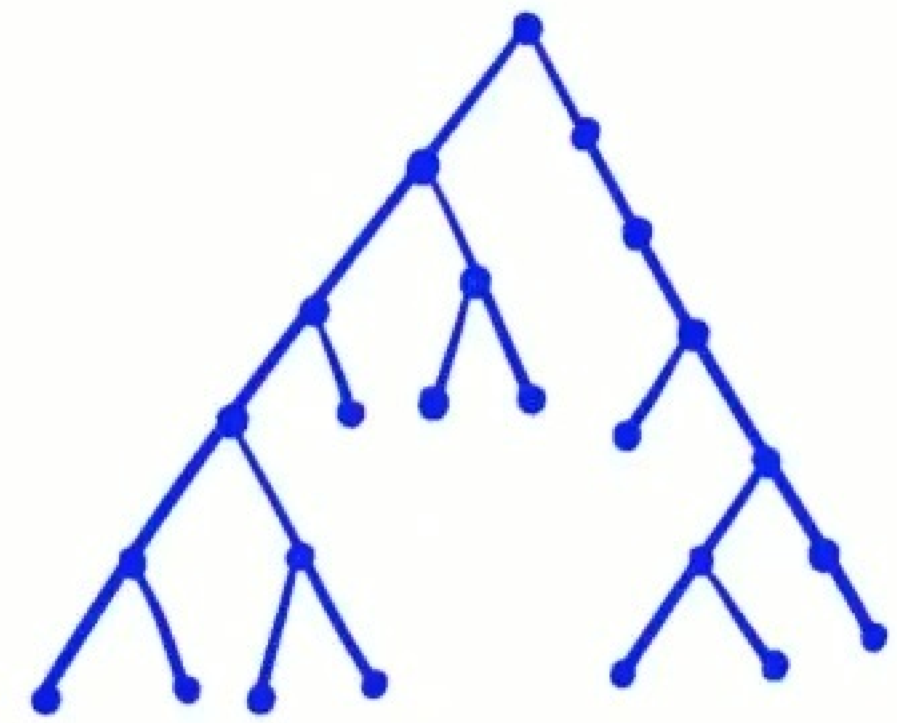

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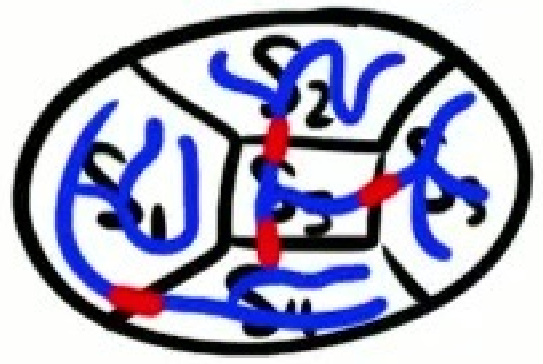

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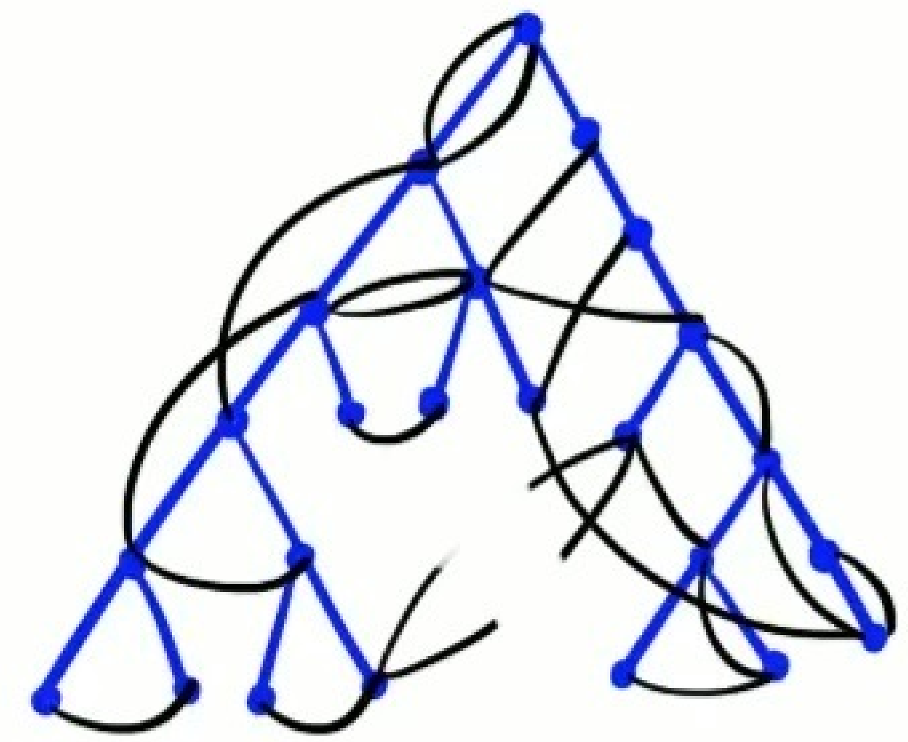

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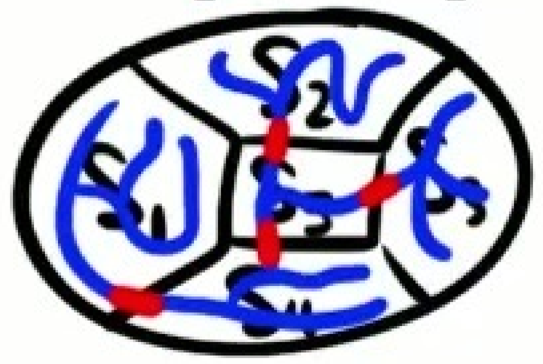

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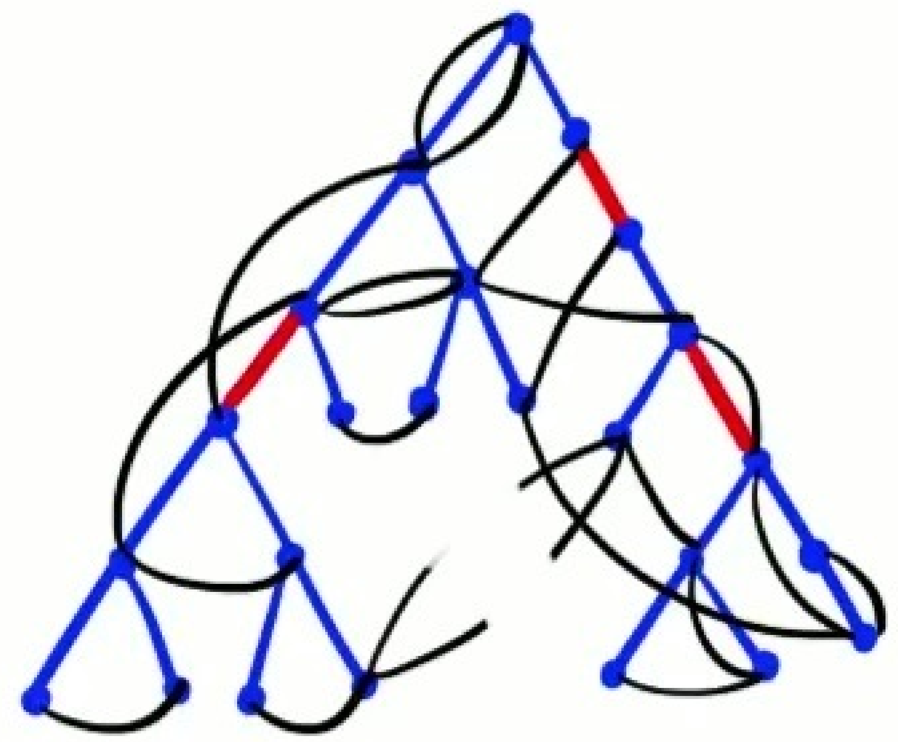

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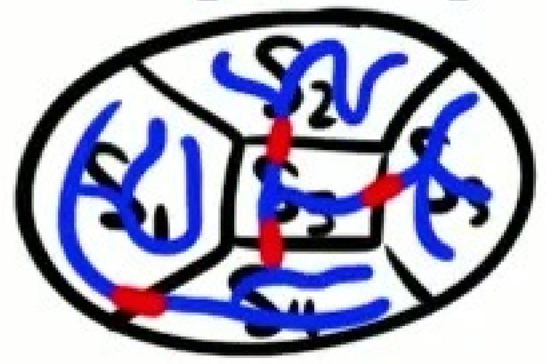

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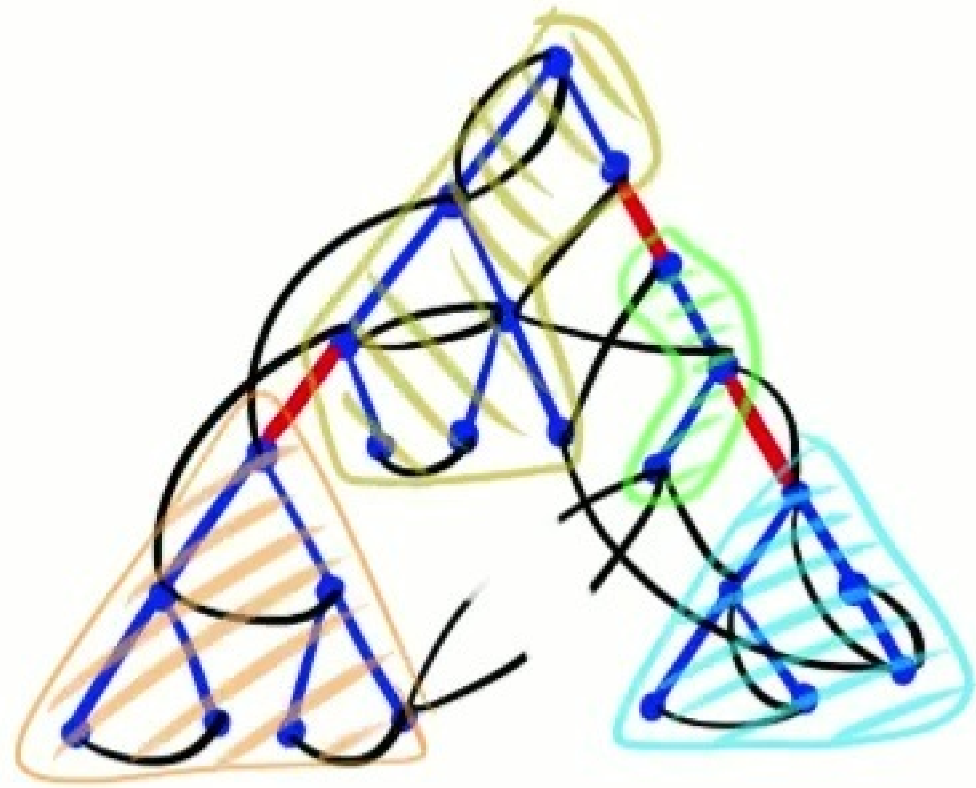

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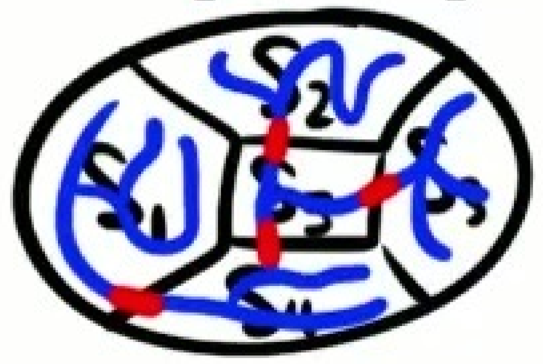

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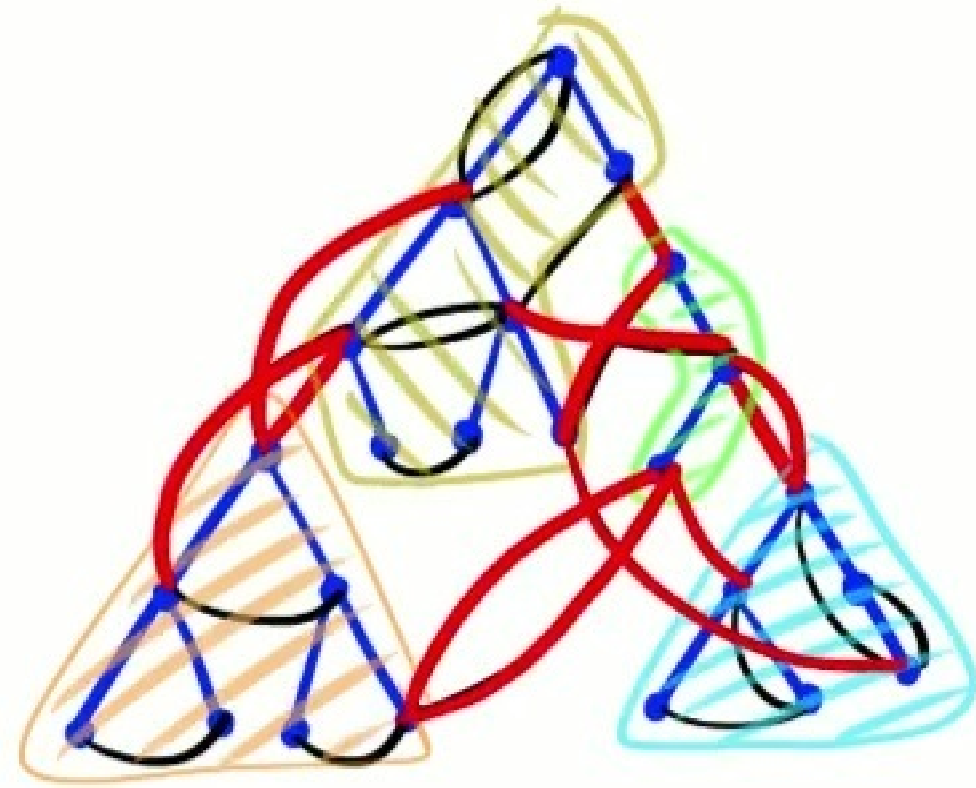

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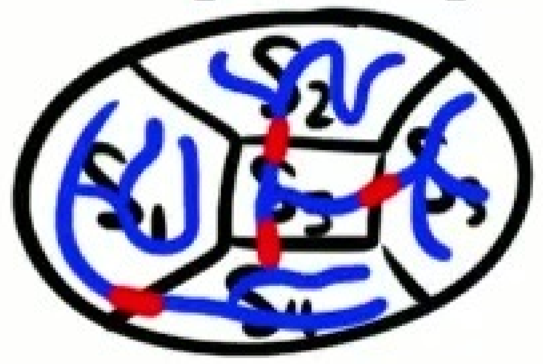

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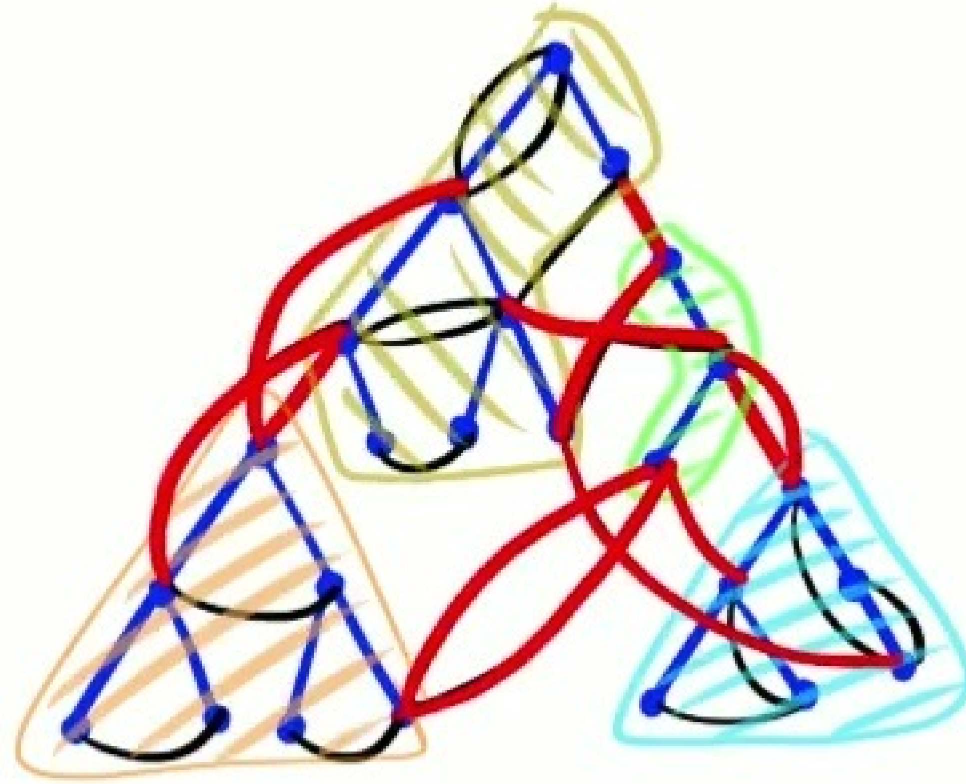

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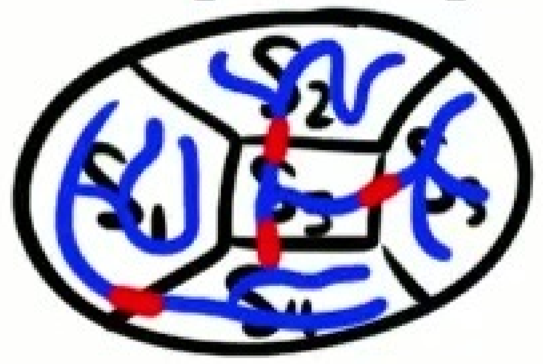
Tree Packing [Thorup]: [GLL'18]:

Given weighted graph, exists collection $\mathcal{T}$ of ~~poly(n)~~ n^k spanning trees of G s.t. for any min k-cut $S_1, S_2, \dots, S_k \subseteq V$ exists tree $T \in \mathcal{T}$ s.t. $|E_T[S_1, S_2, \dots, S_k]| \le \cancel{2k-2}\ k-1$

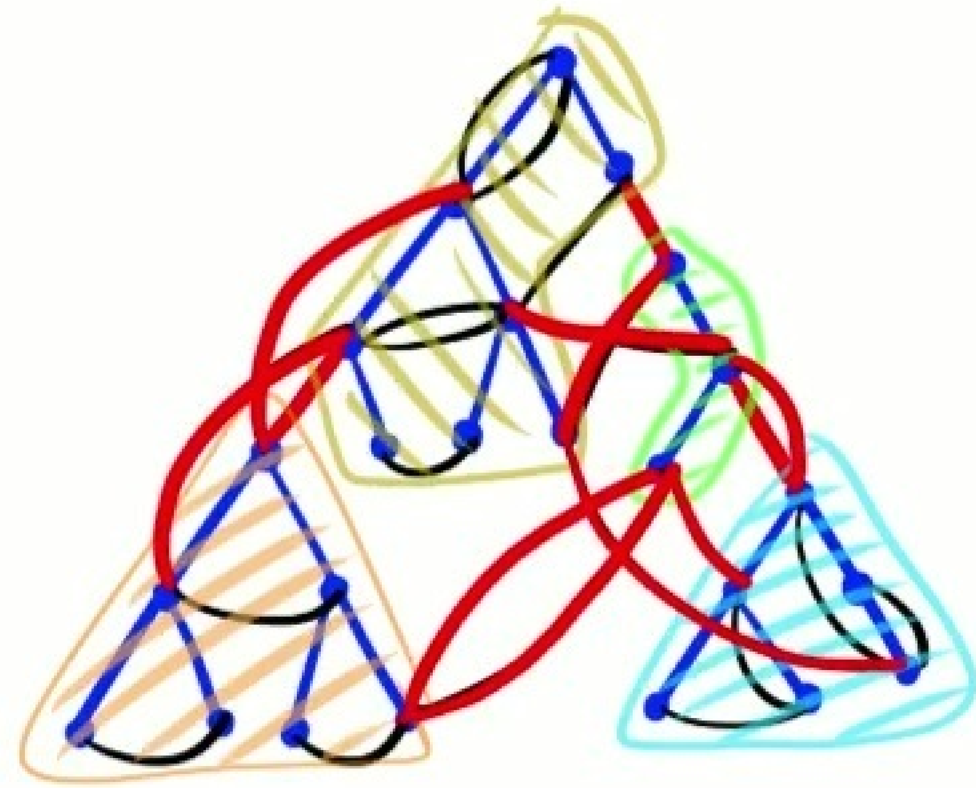


Suffices to solve problem: given (tight) tree T, delete best k-1 edges to form smallest k-cut

Time $\tilde{O}(\lambda_k^k n^{o(k)})$

λ_k^k is FPT in λ_k and k,

but particular dependency matters!



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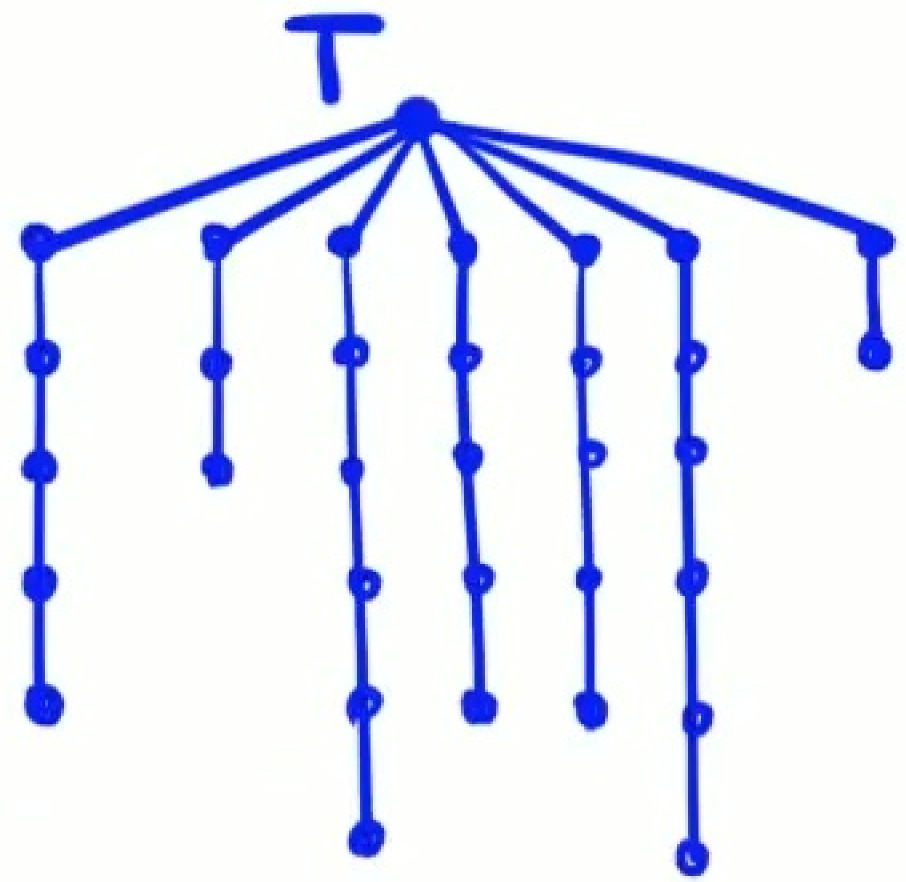

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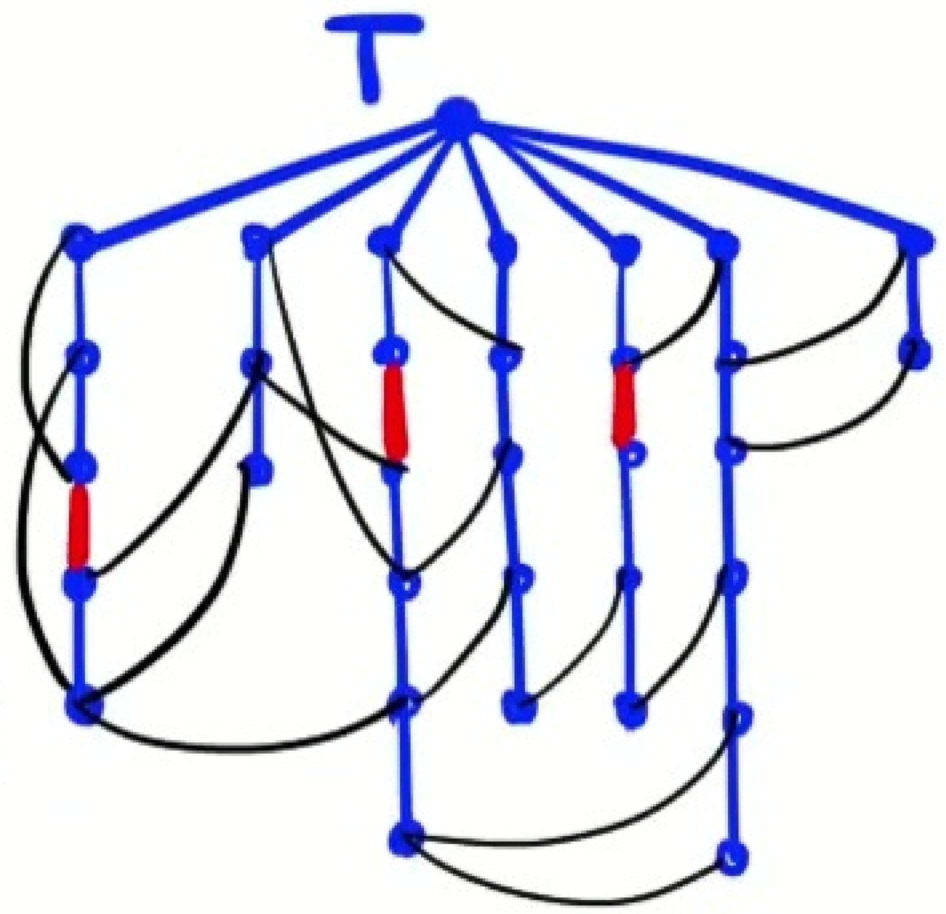

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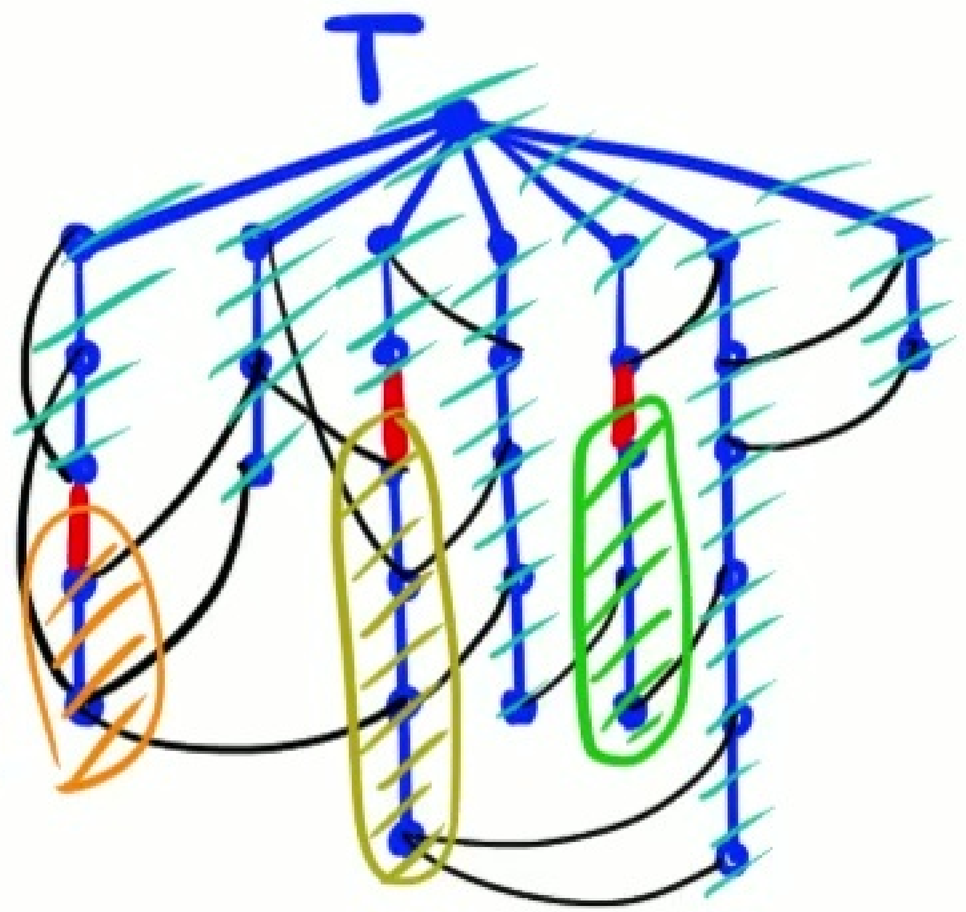

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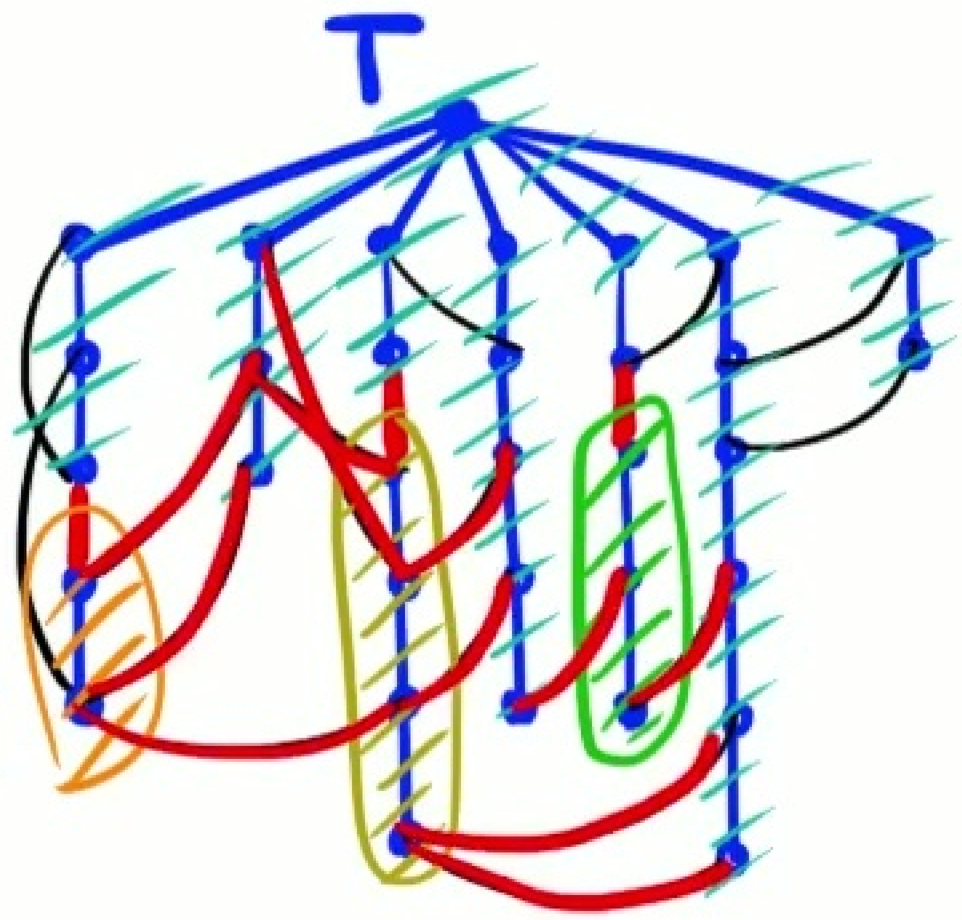

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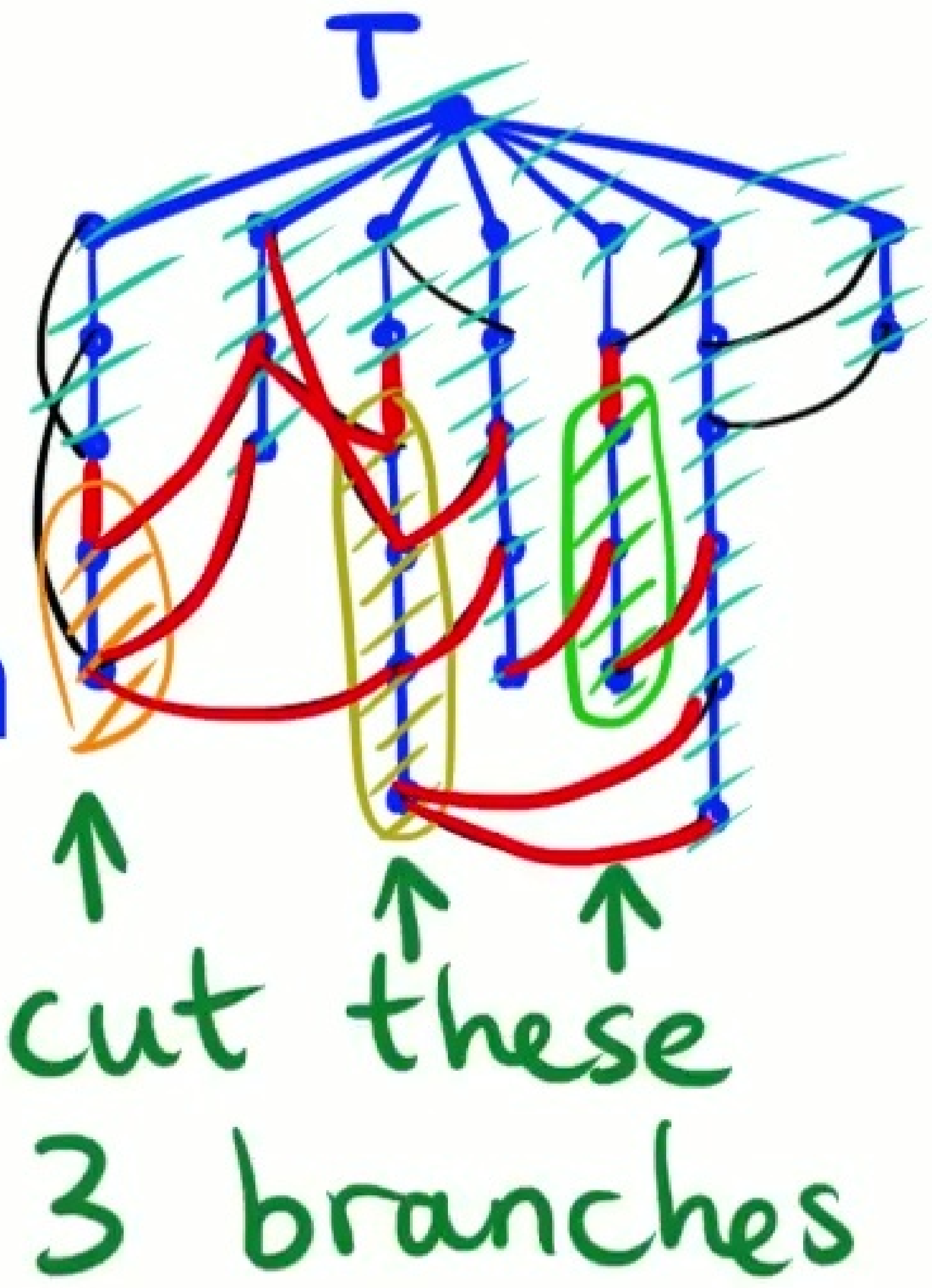

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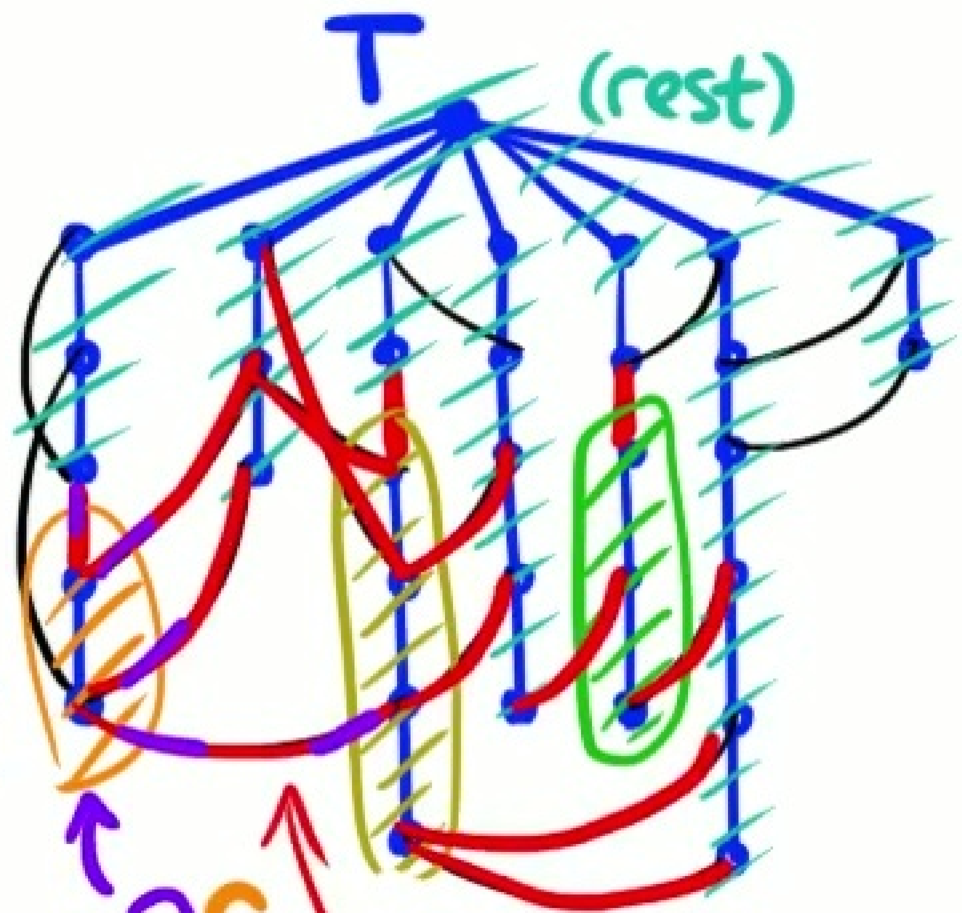

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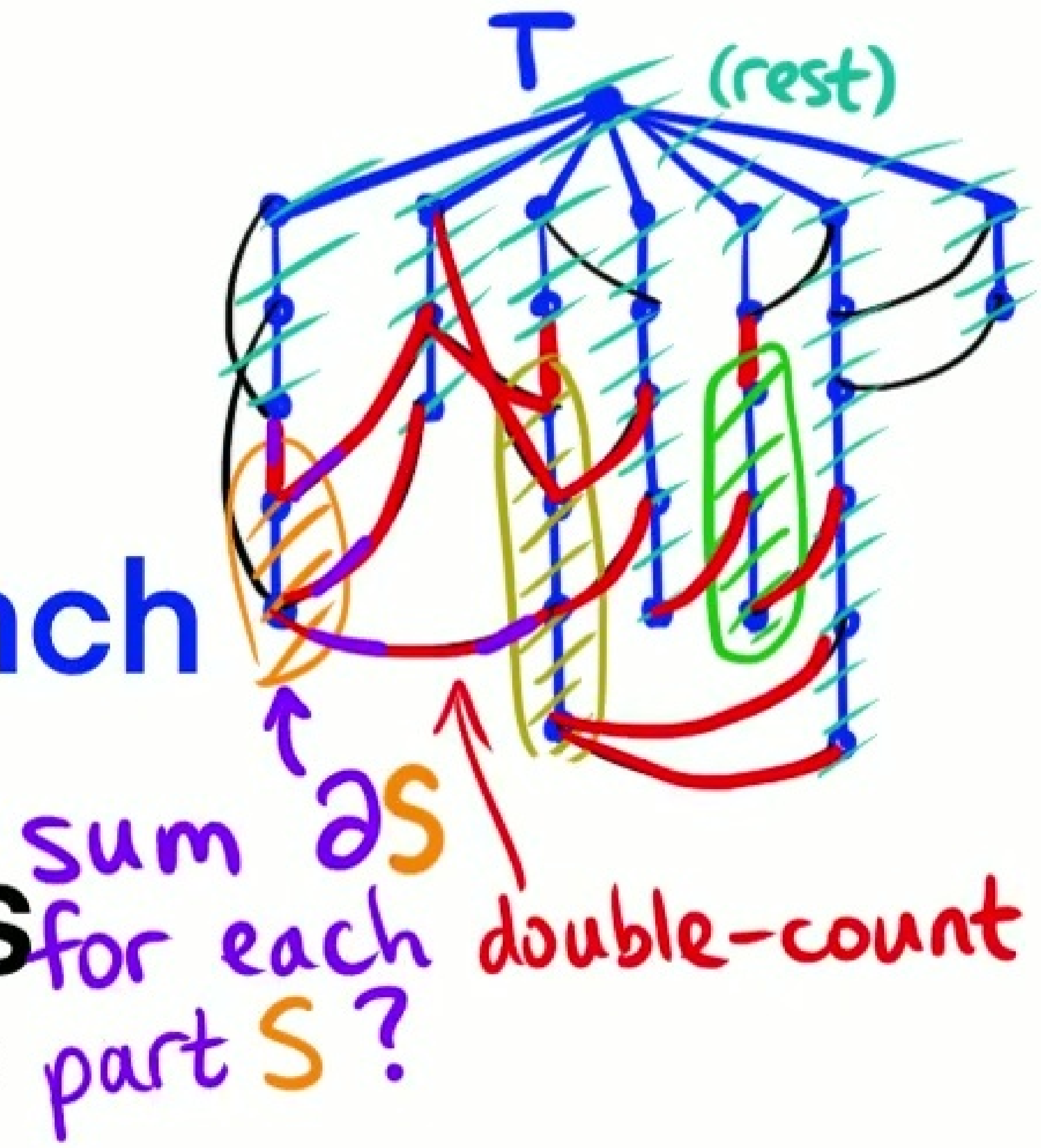

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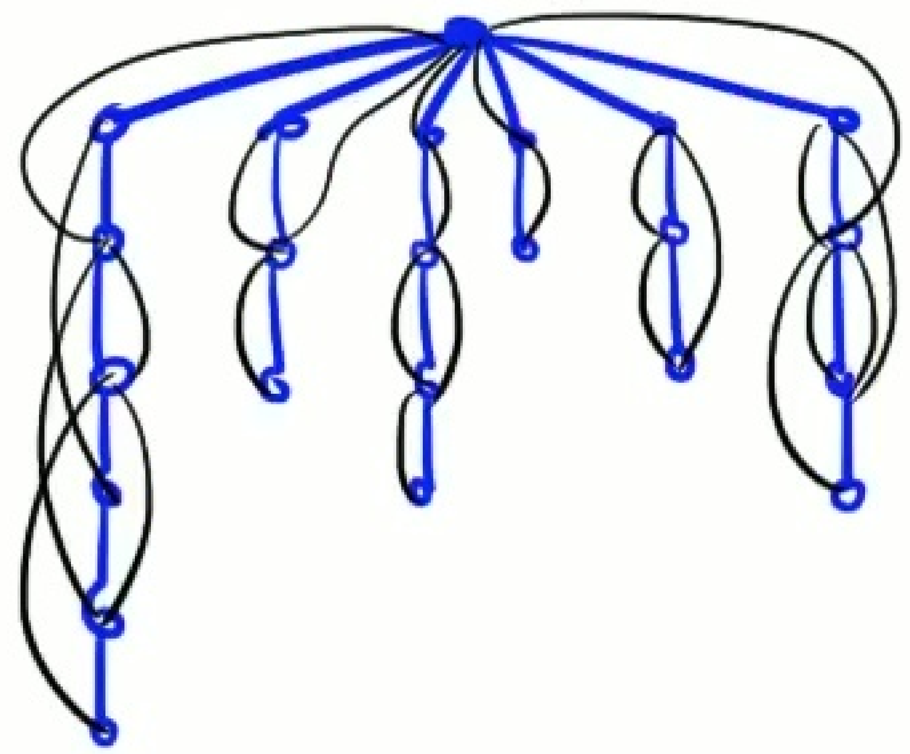
② Extreme case: no edges between different branches



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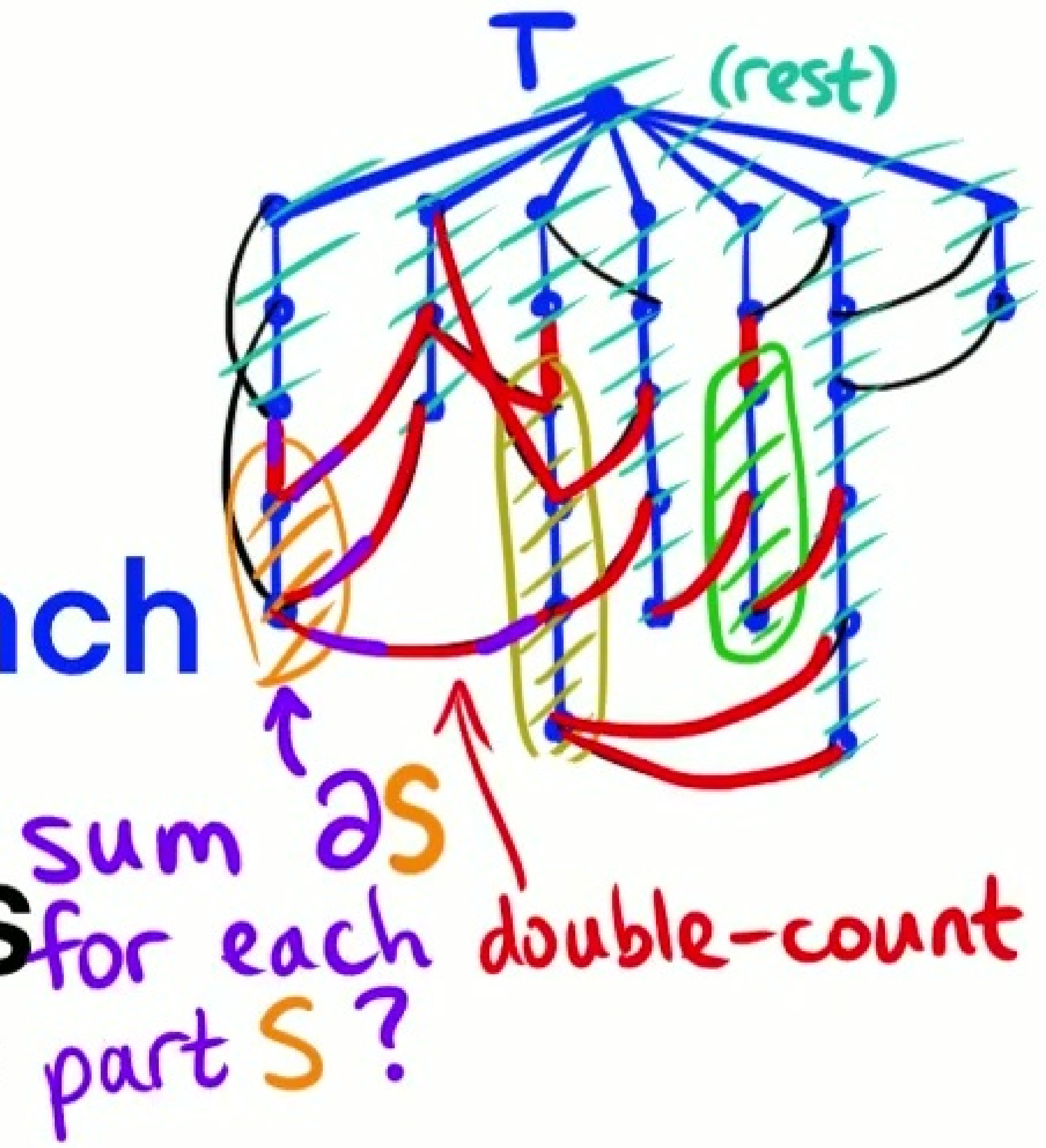

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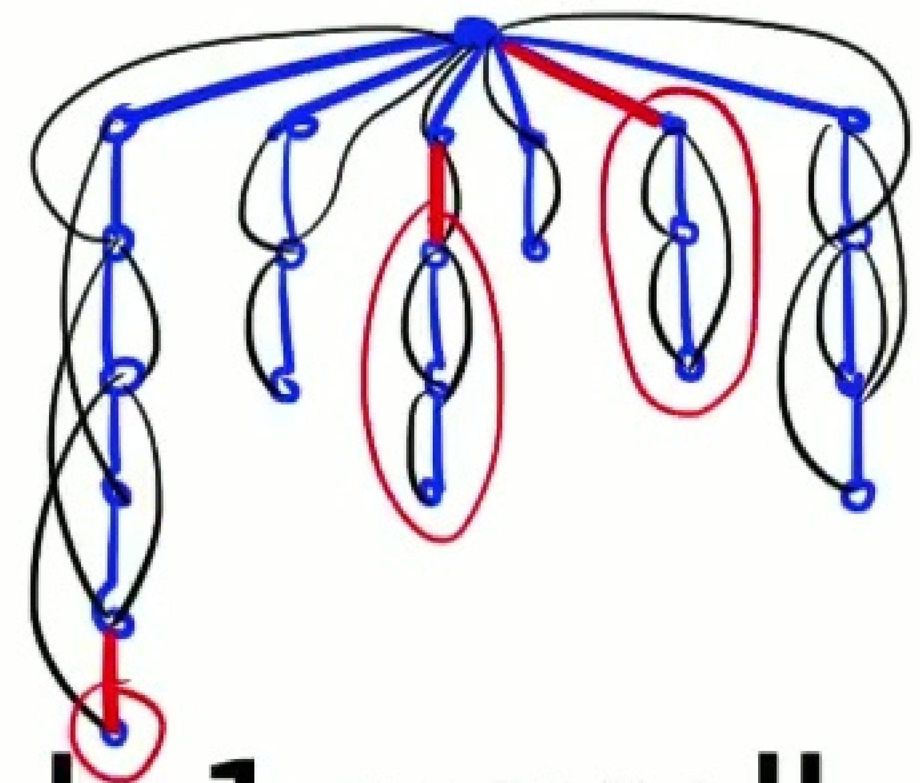

② Extreme case: no edges between different branches

For each branch, take best cut; take best k-1 overall

Identifying the Branches

Other extreme: suppose many edges between OPT's branches

Identifying the Branches

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Idea: color-code a spanning tree connecting these branches

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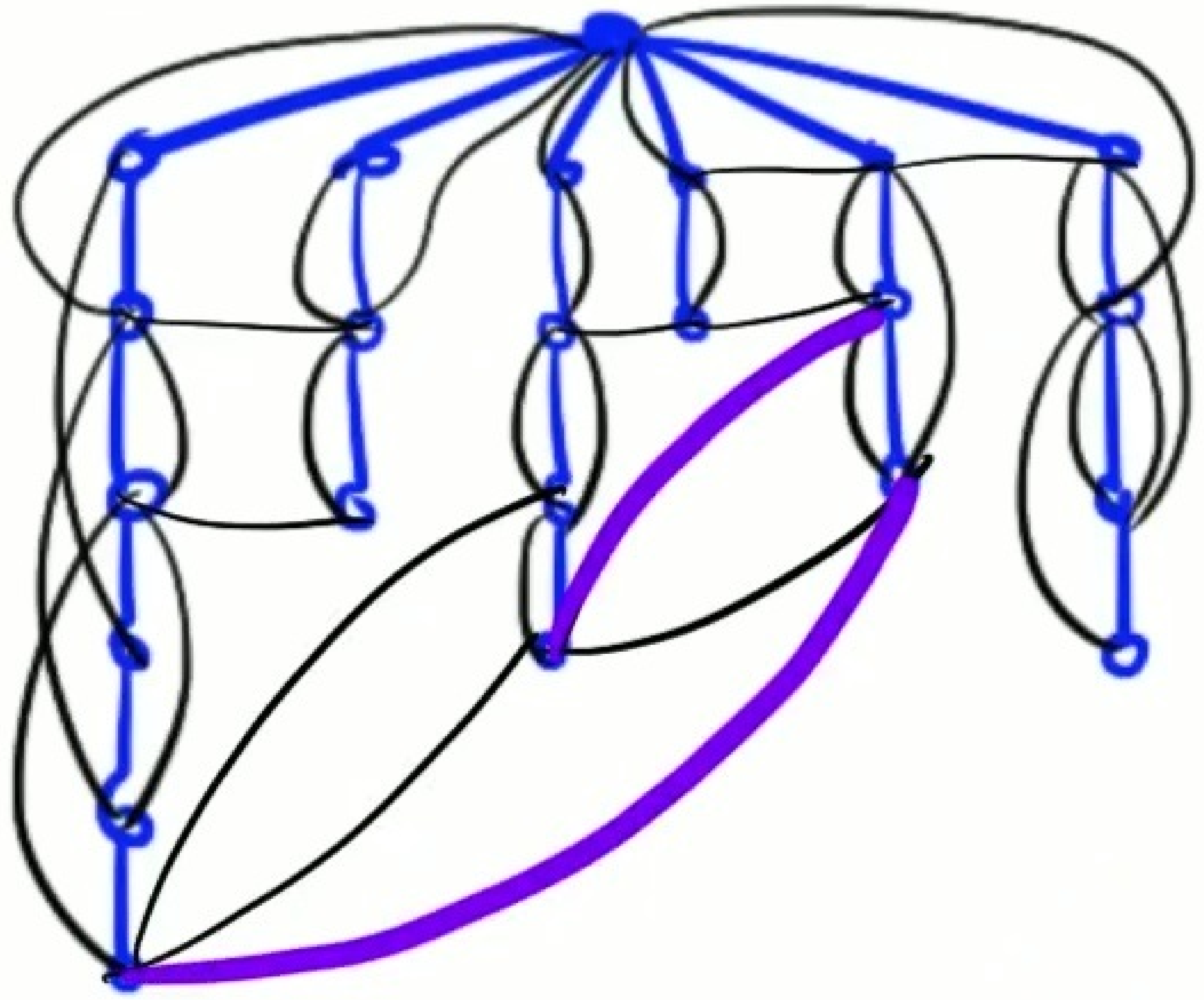

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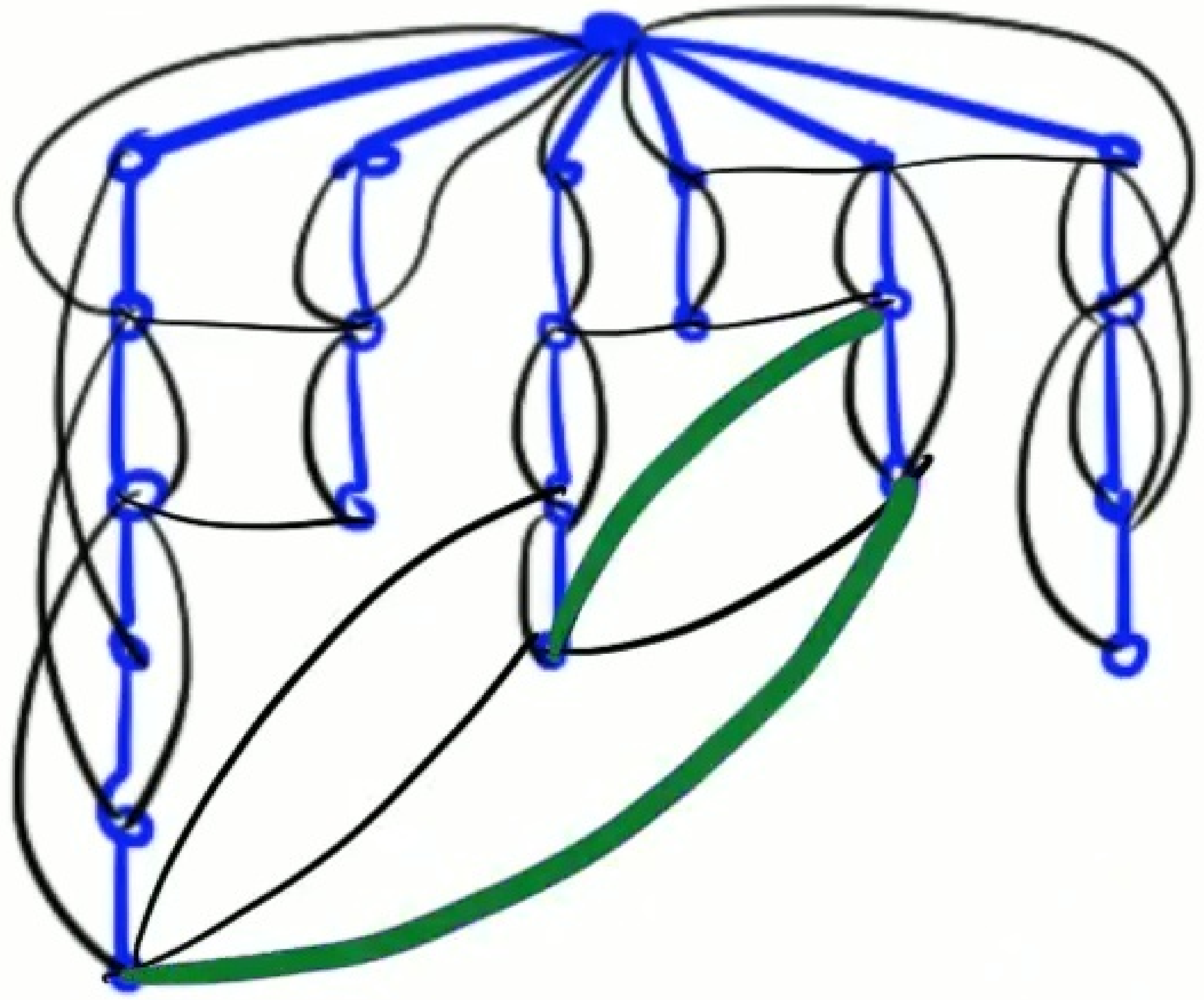

Color each edge green w.p. p and red w.p. 1-p

Want: fixed spanning tree all green

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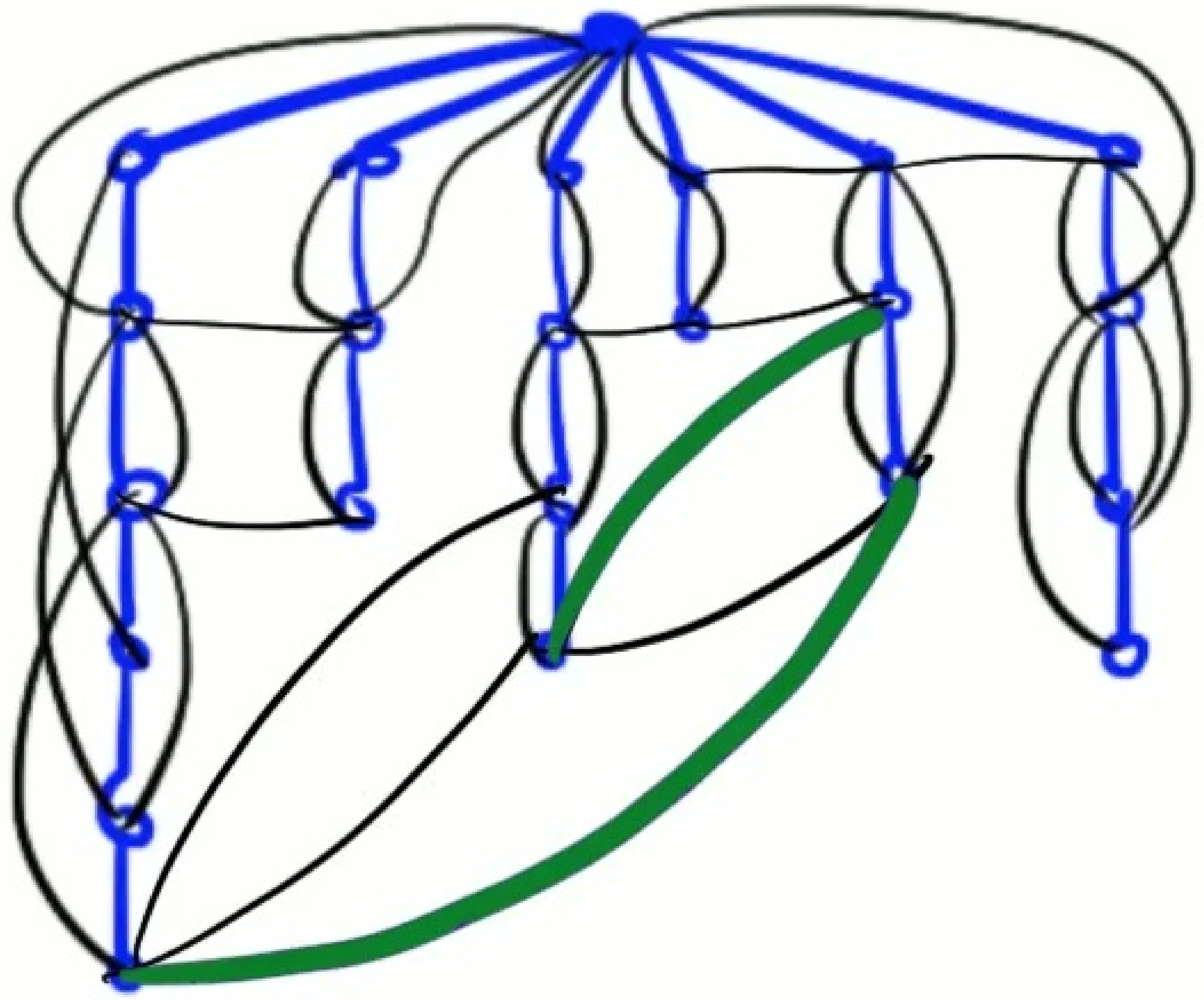

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For each of OPT's branches, remaining boundary edges red

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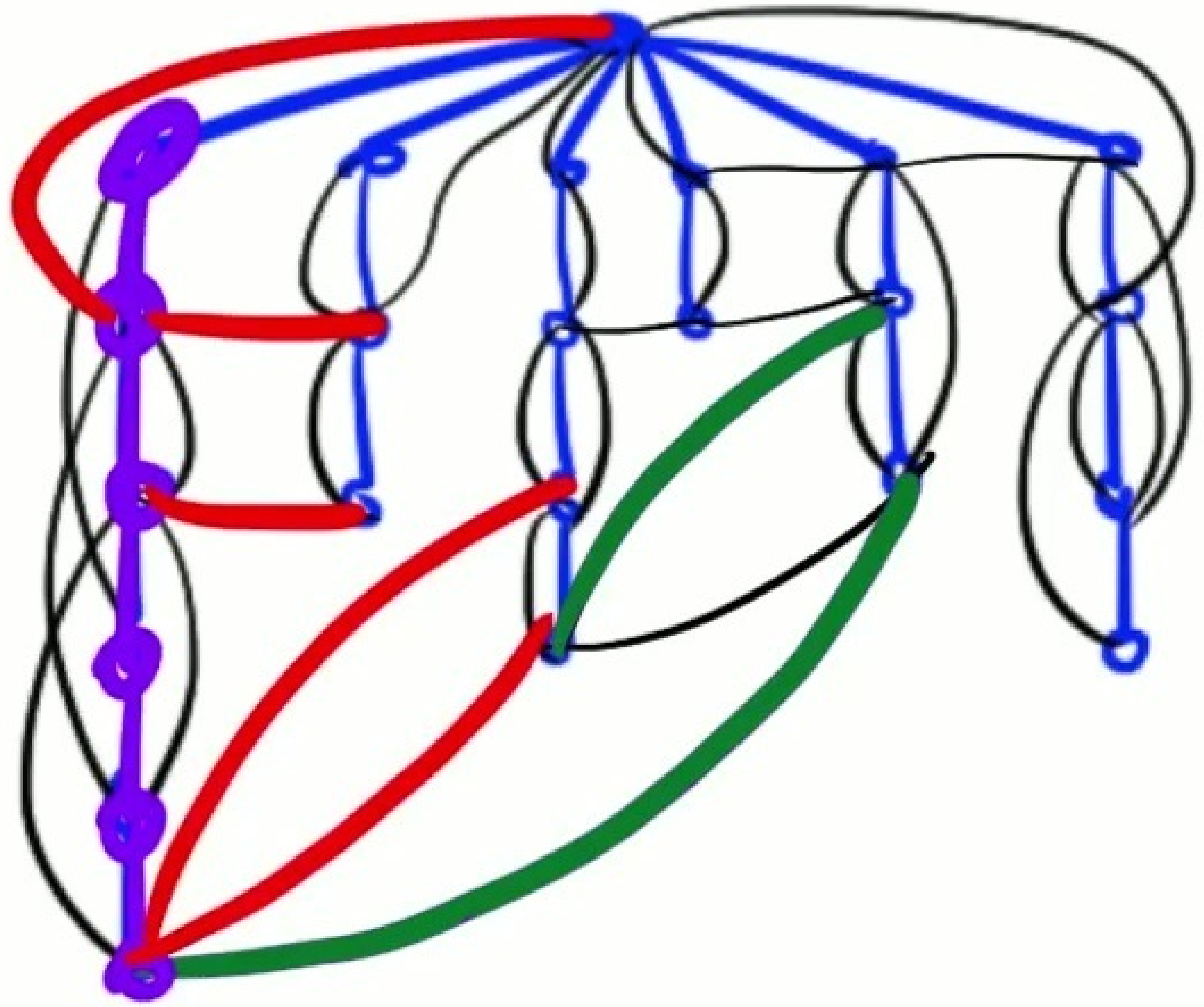

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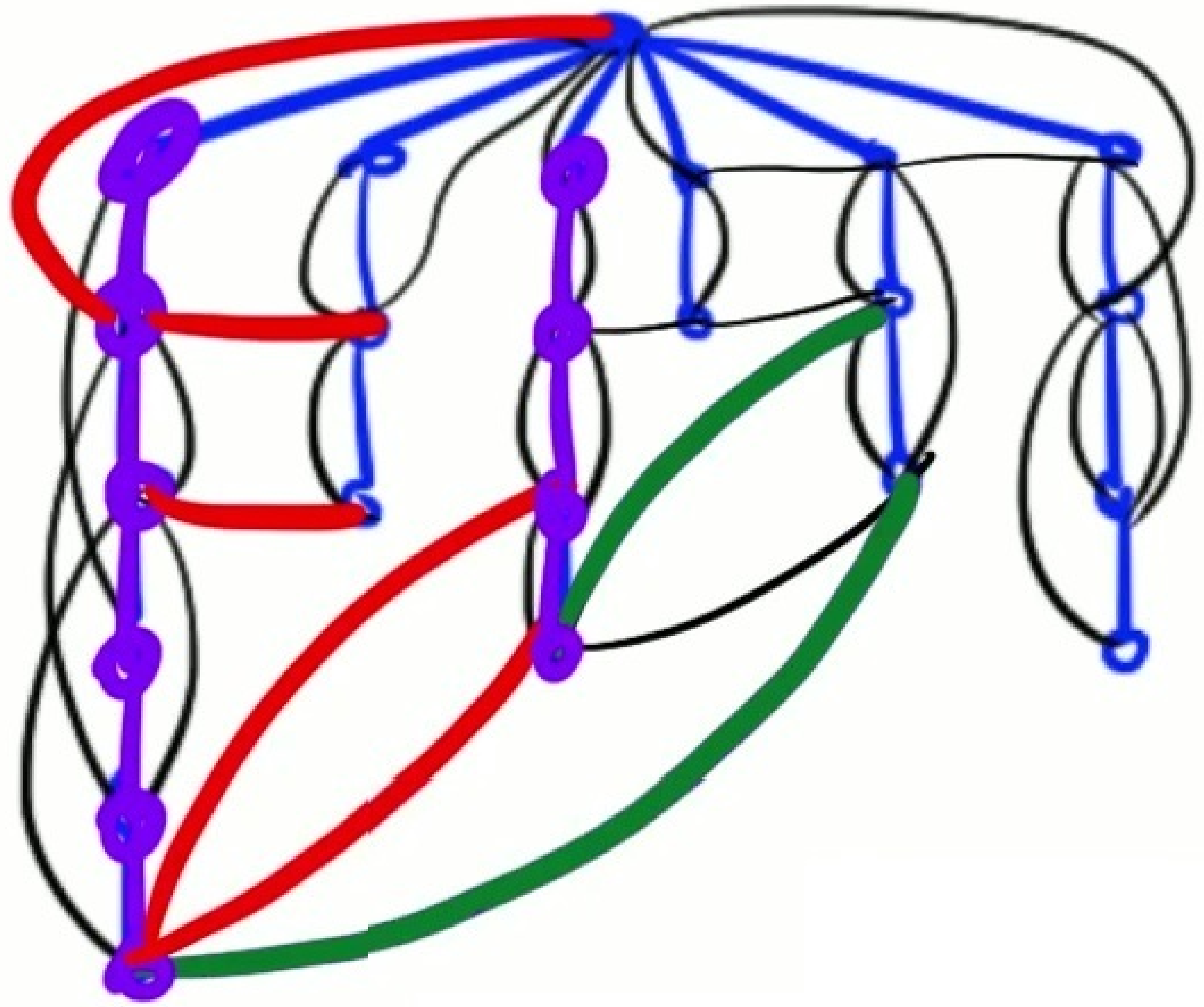

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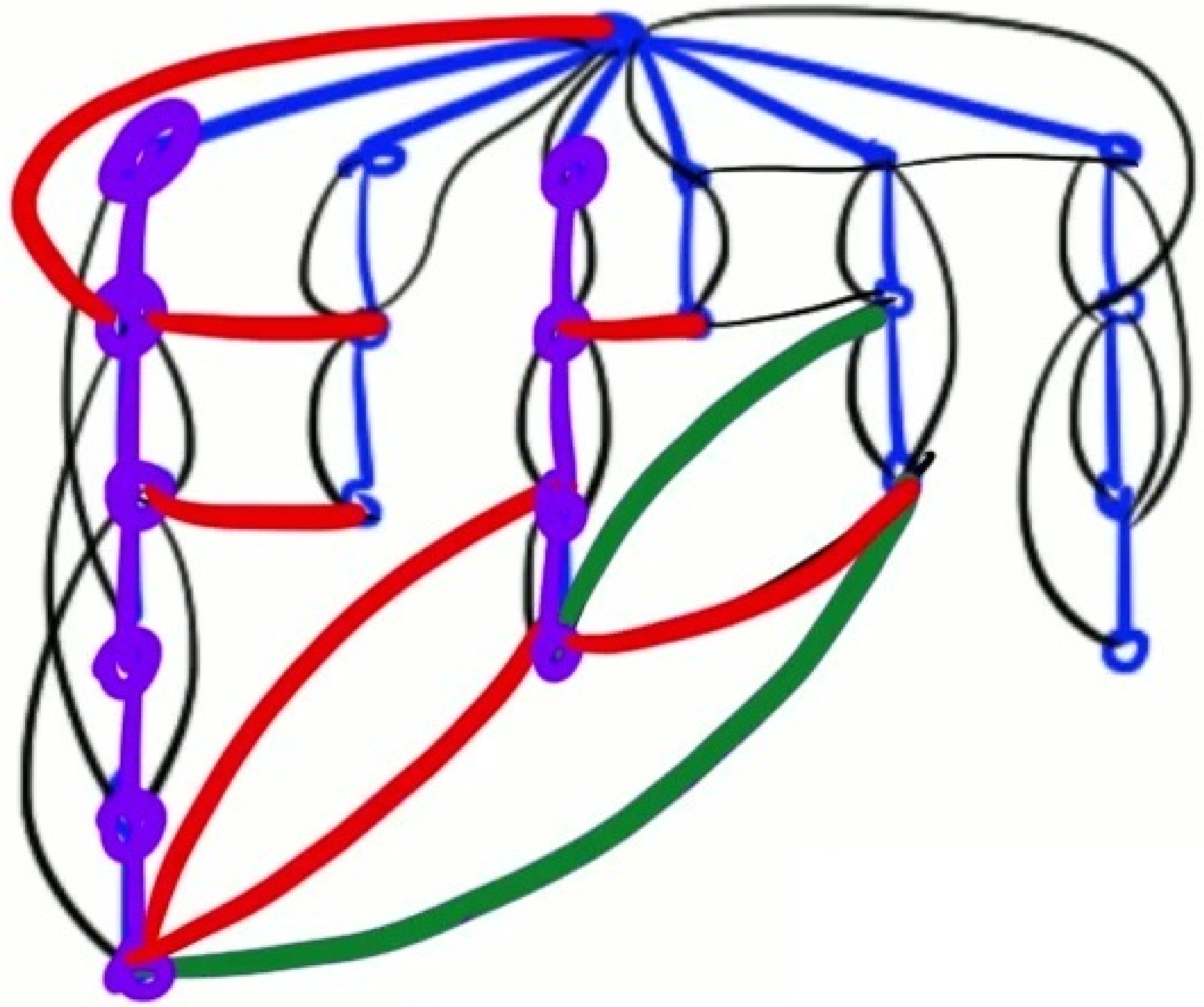

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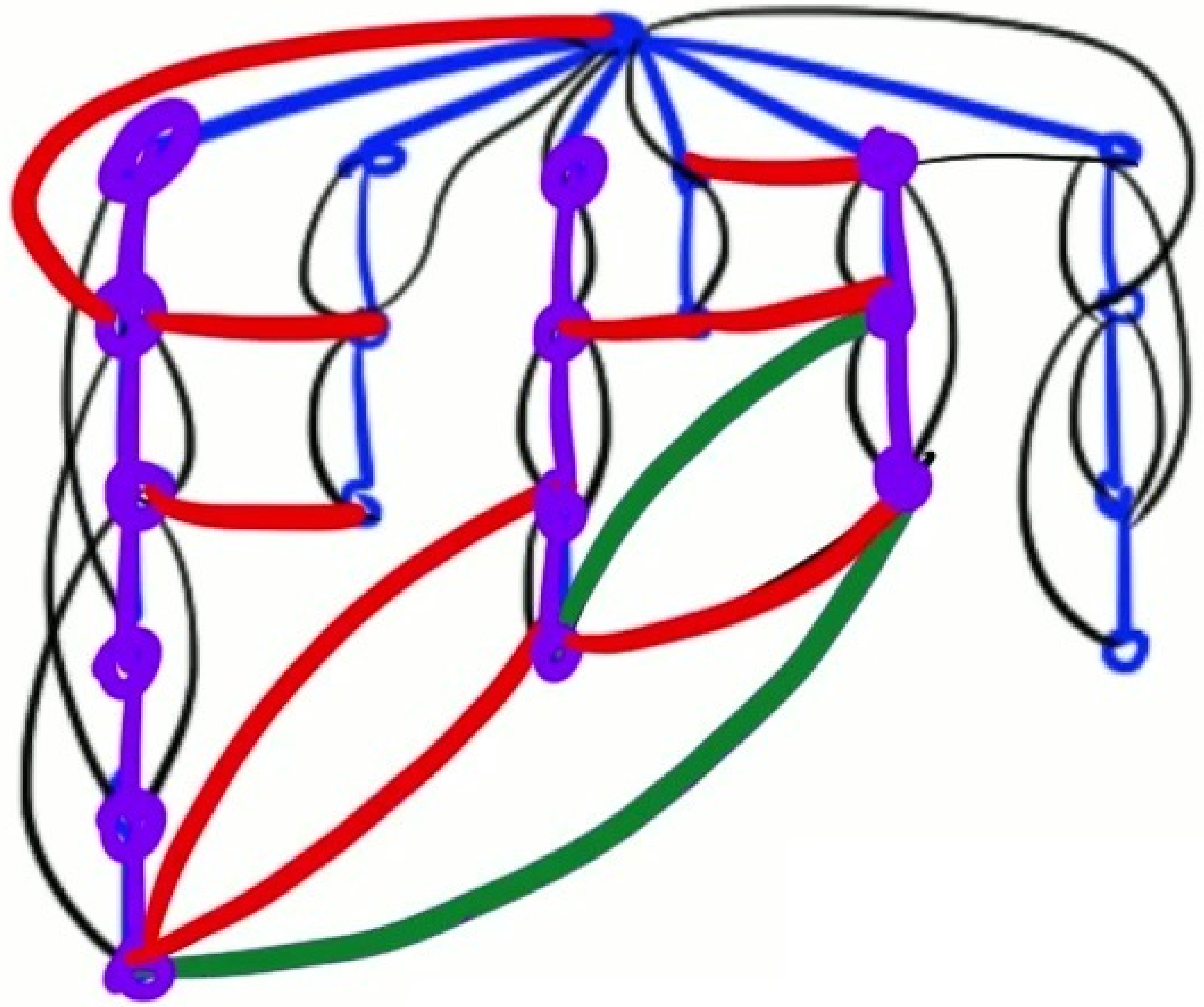

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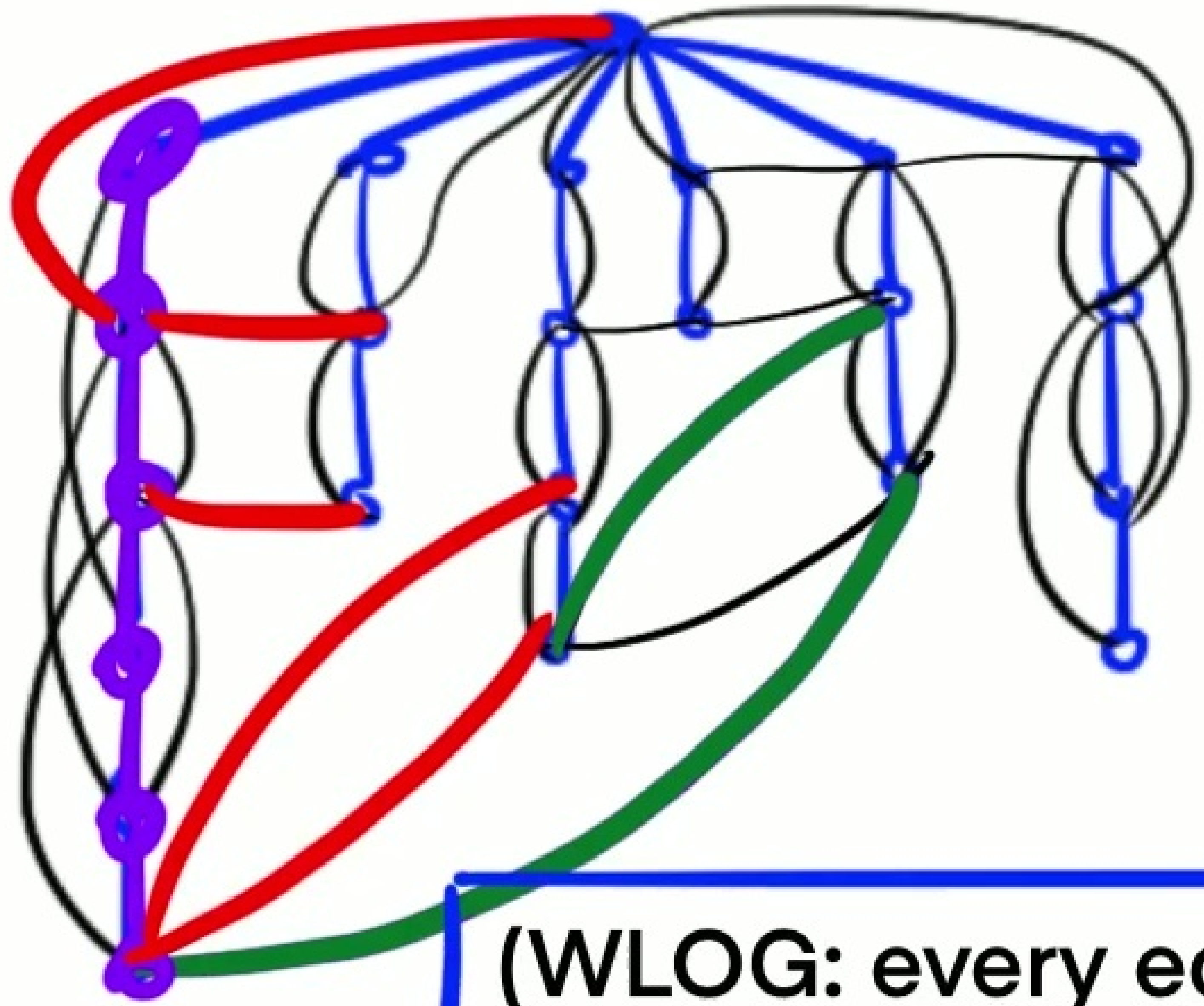

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(WLOG: every edge cut $\leq \lambda_k$
Otherwise OPT can't pick that edge, so contract)

For each of OPT's branches, remaining boundary edges red $\leq \lambda_k$ per branch $\Rightarrow \leq k\lambda_k$ total

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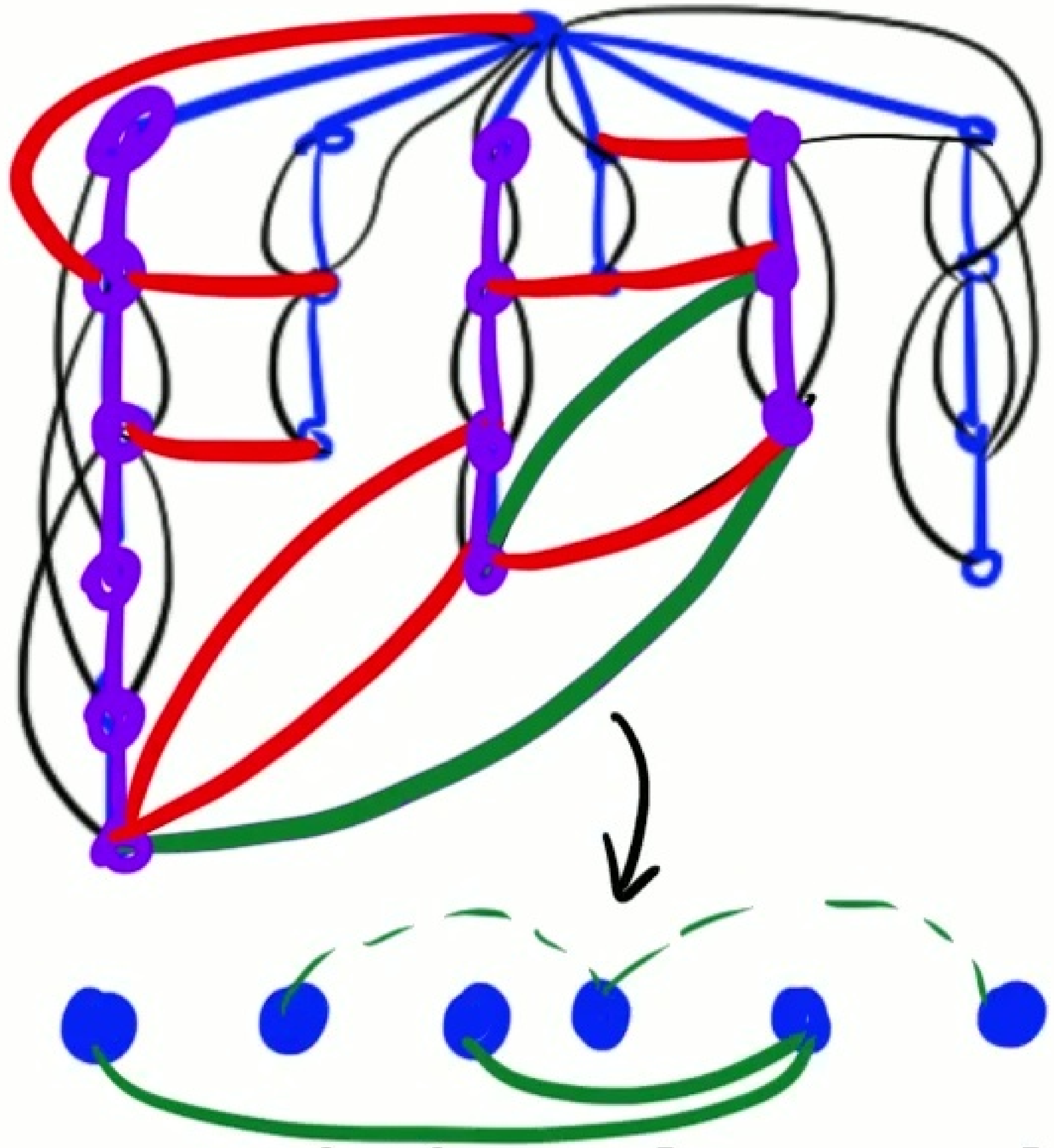

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Contract each branch; OPT's branches is one connected component

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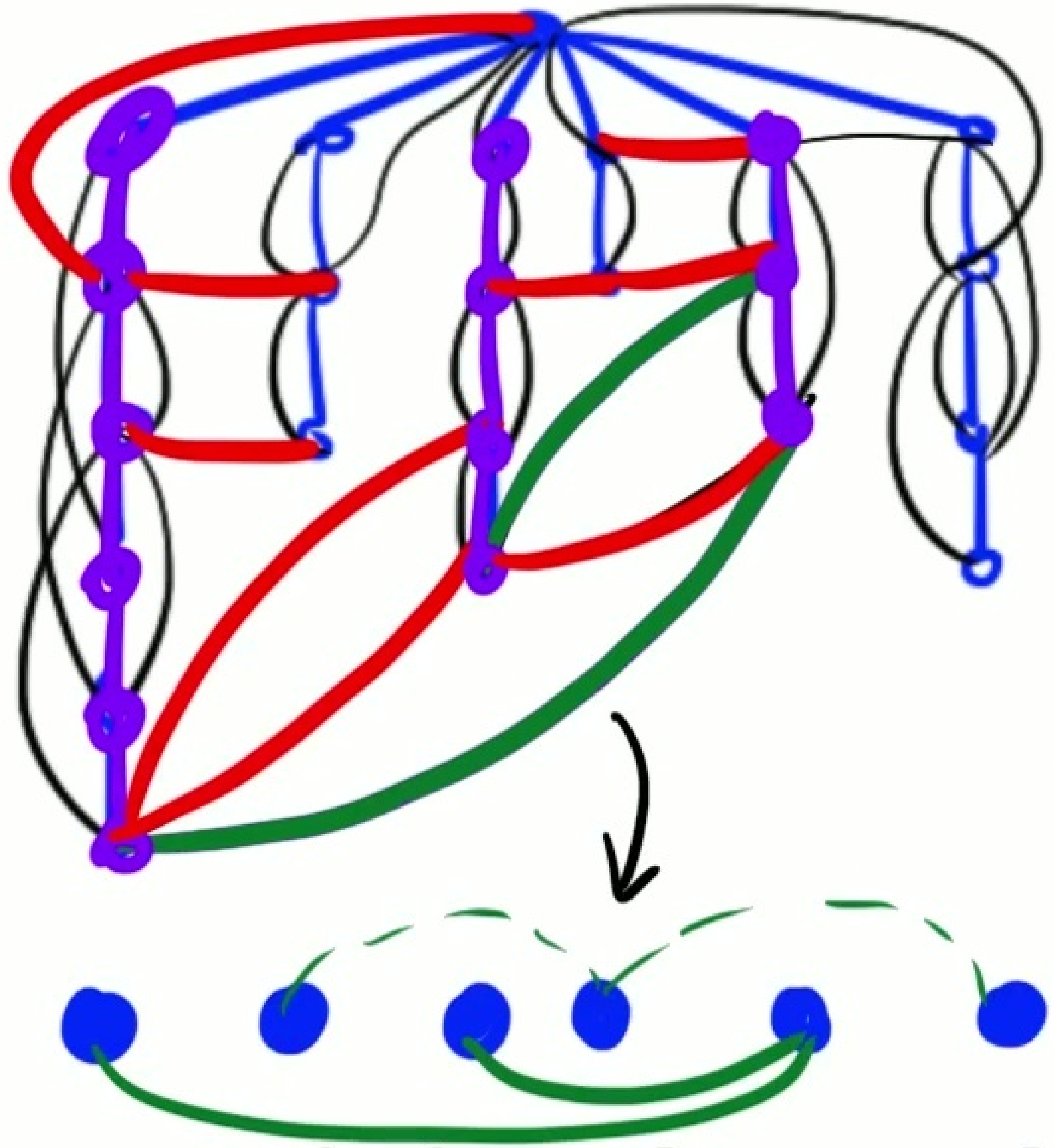

success $p^k(1-p)^{k\lambda_k}$

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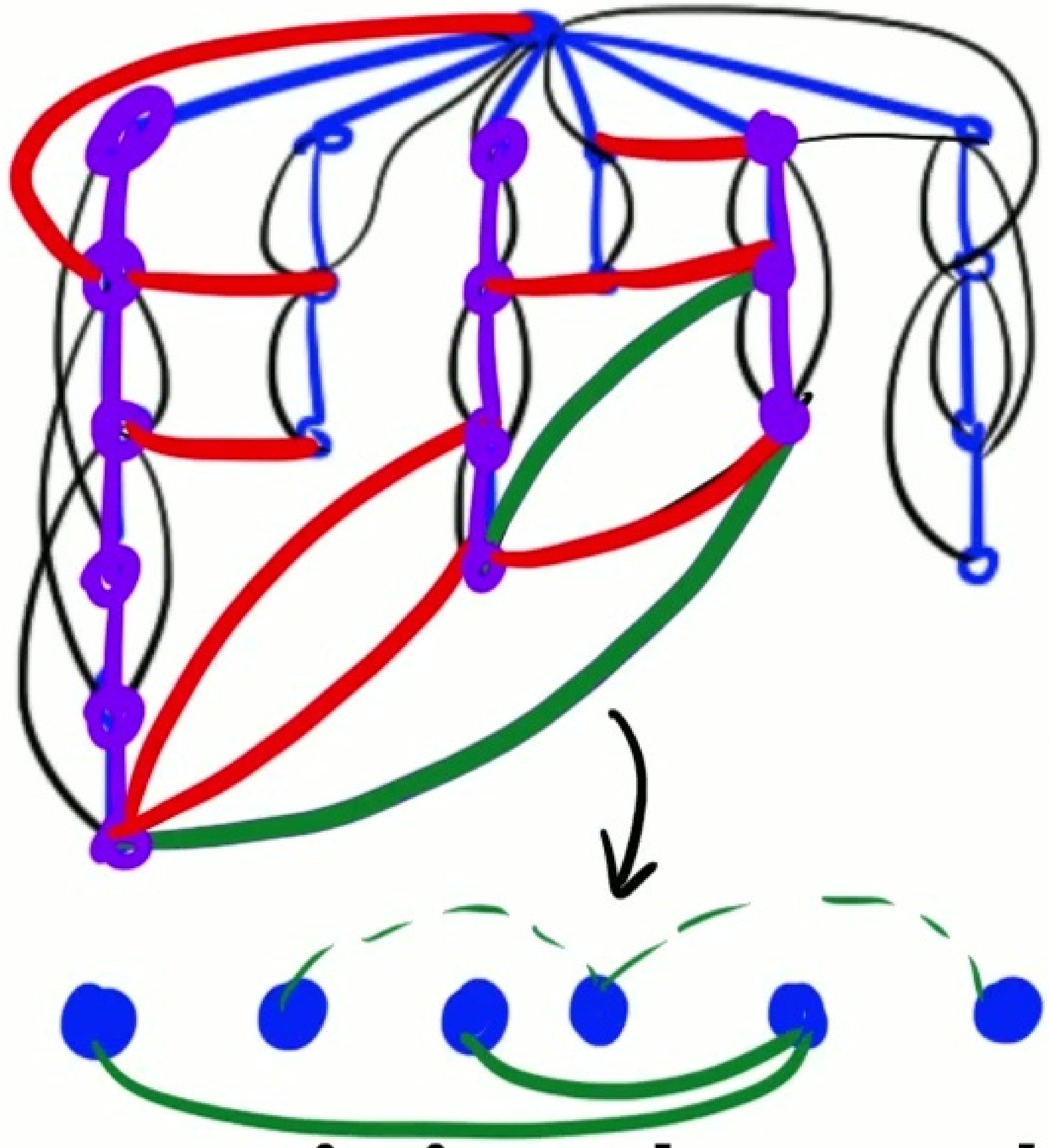

success

$p^k (1-p)^{k\lambda_k}$

$p = 1/\lambda_k$

$\frac{1}{\lambda_k^k} e^{-k}$

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bran...

Highlight small (size k) structure

Idea... e

conn...

Color each e... g... w.p. p and

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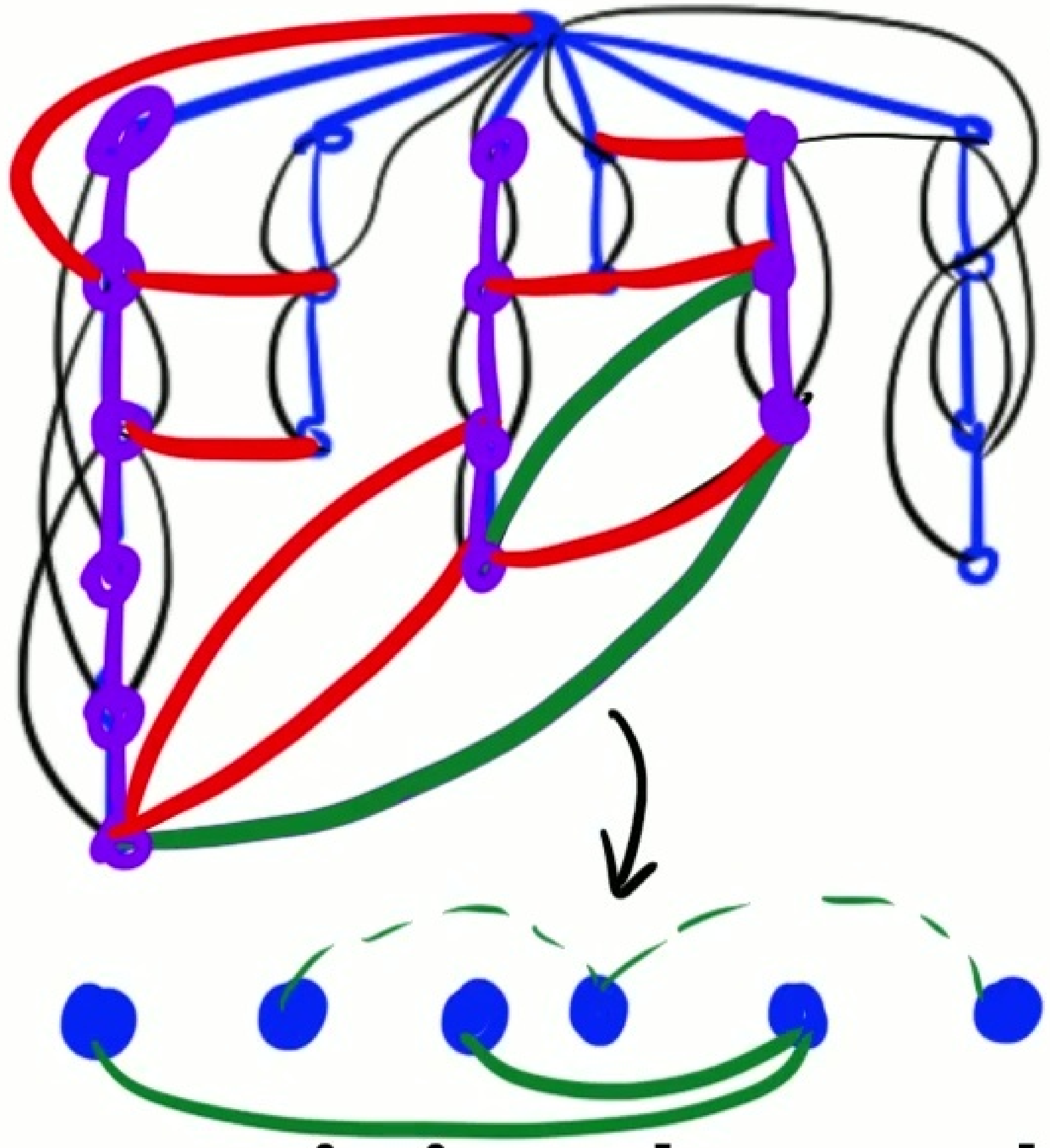

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General Trees

Heavy-light decomposition into branches, color-code red/blue

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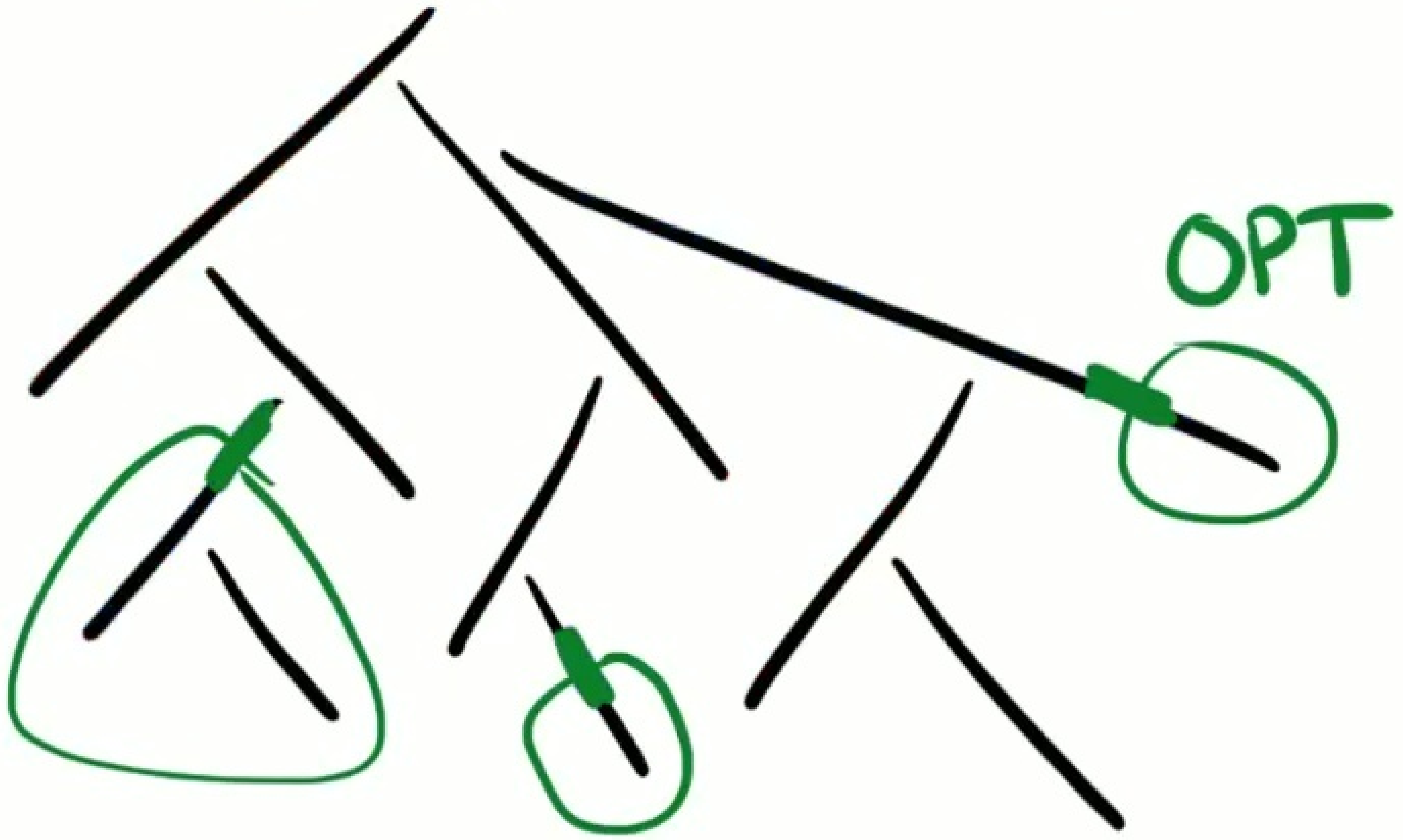

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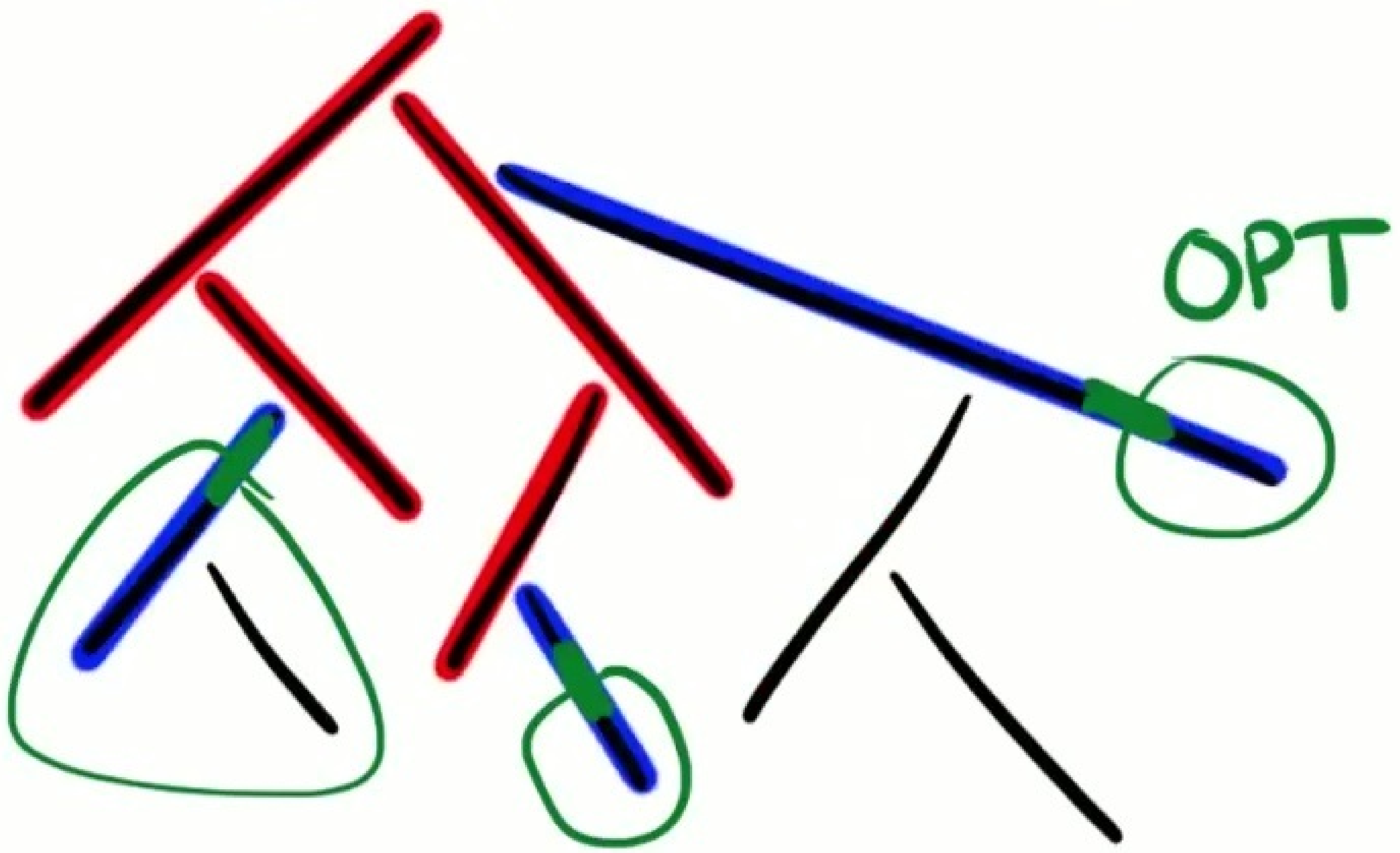

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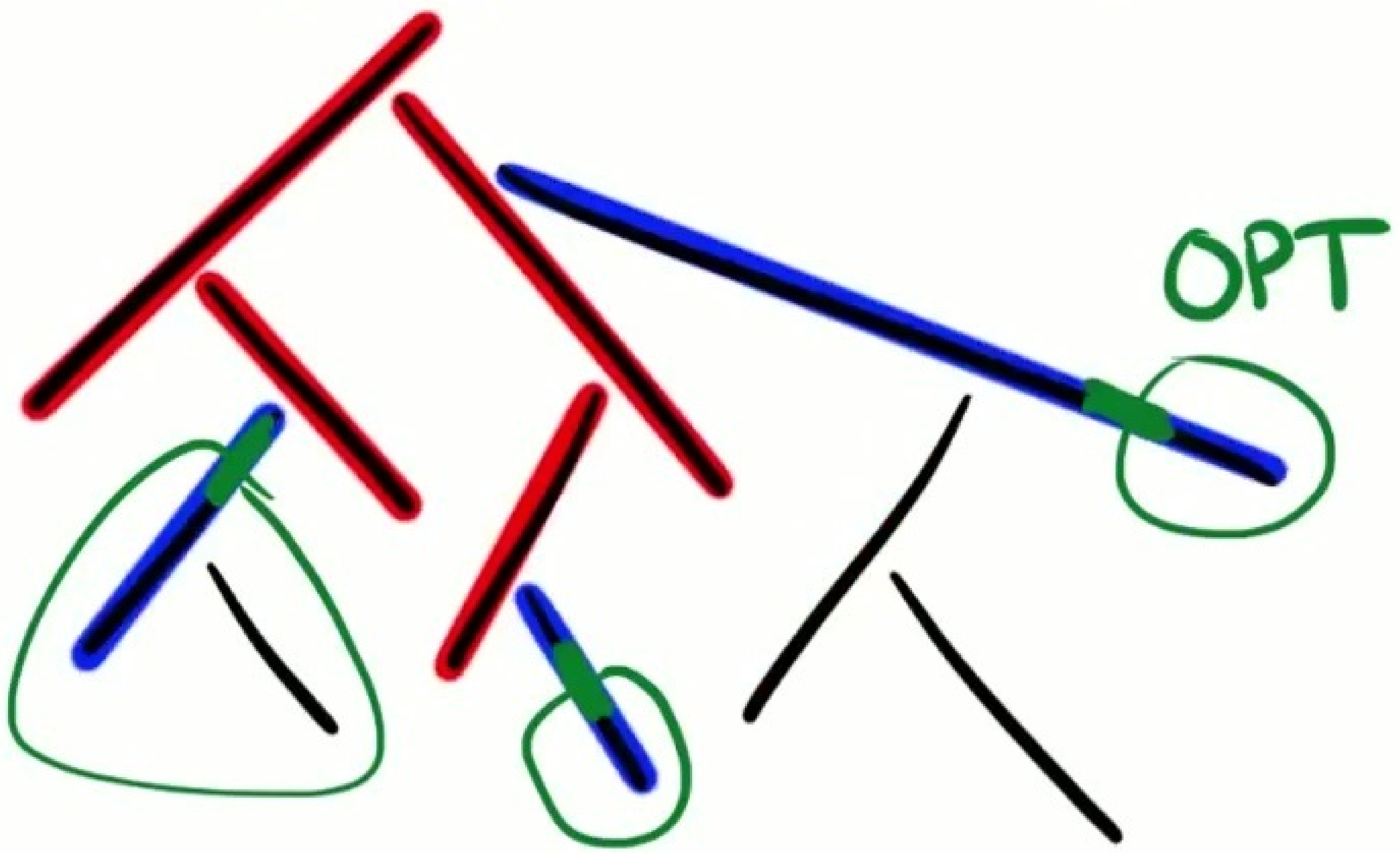

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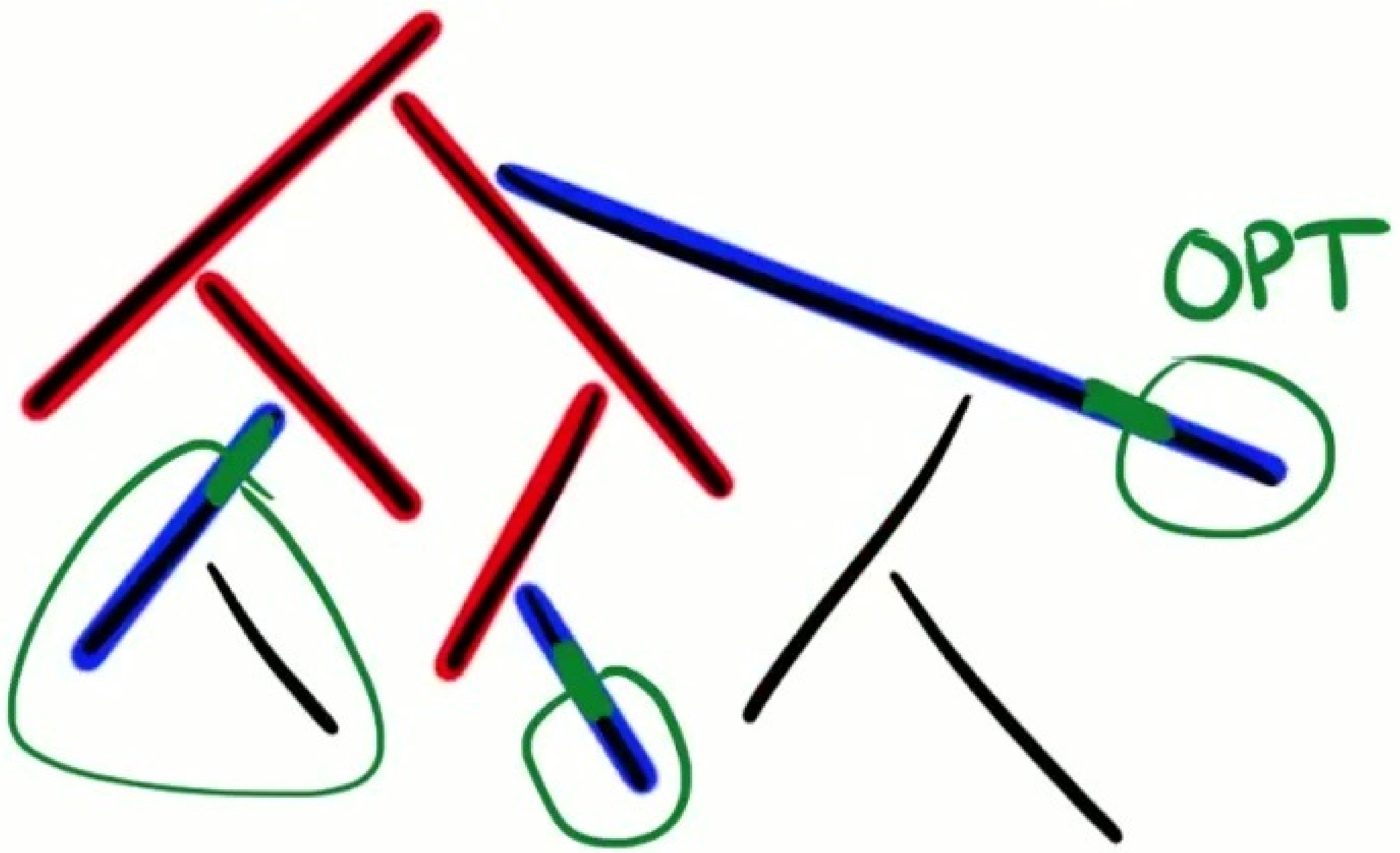

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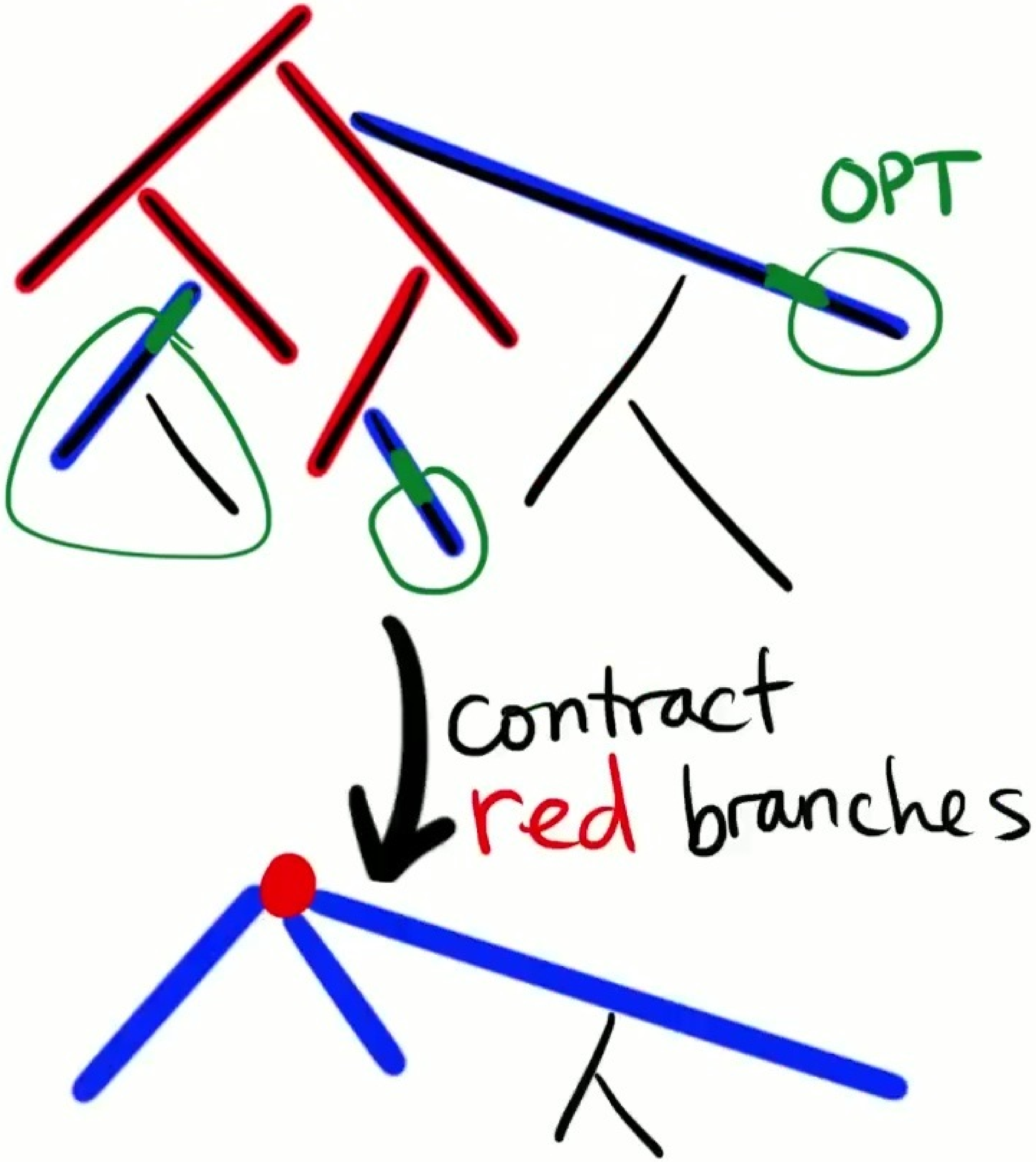

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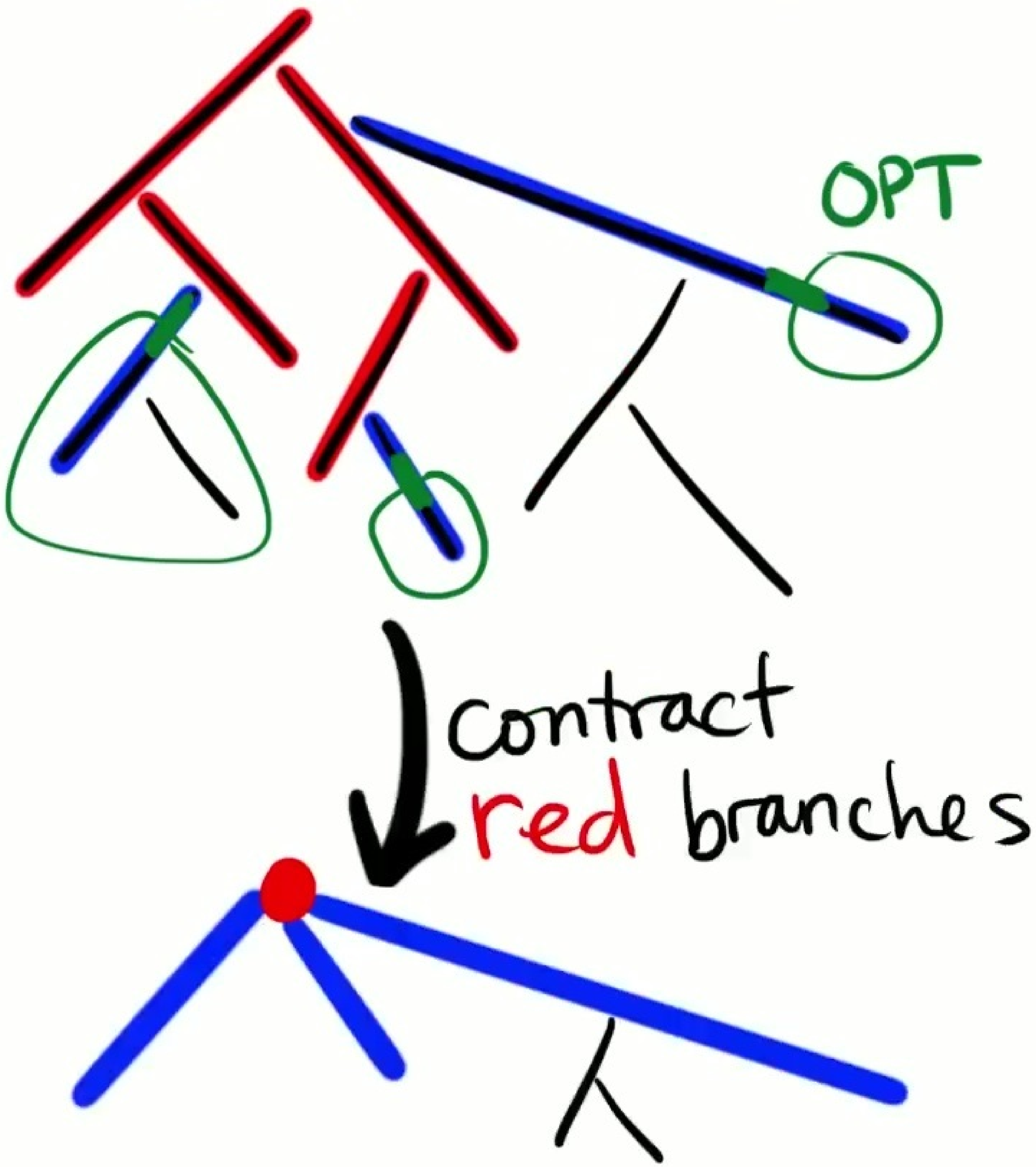

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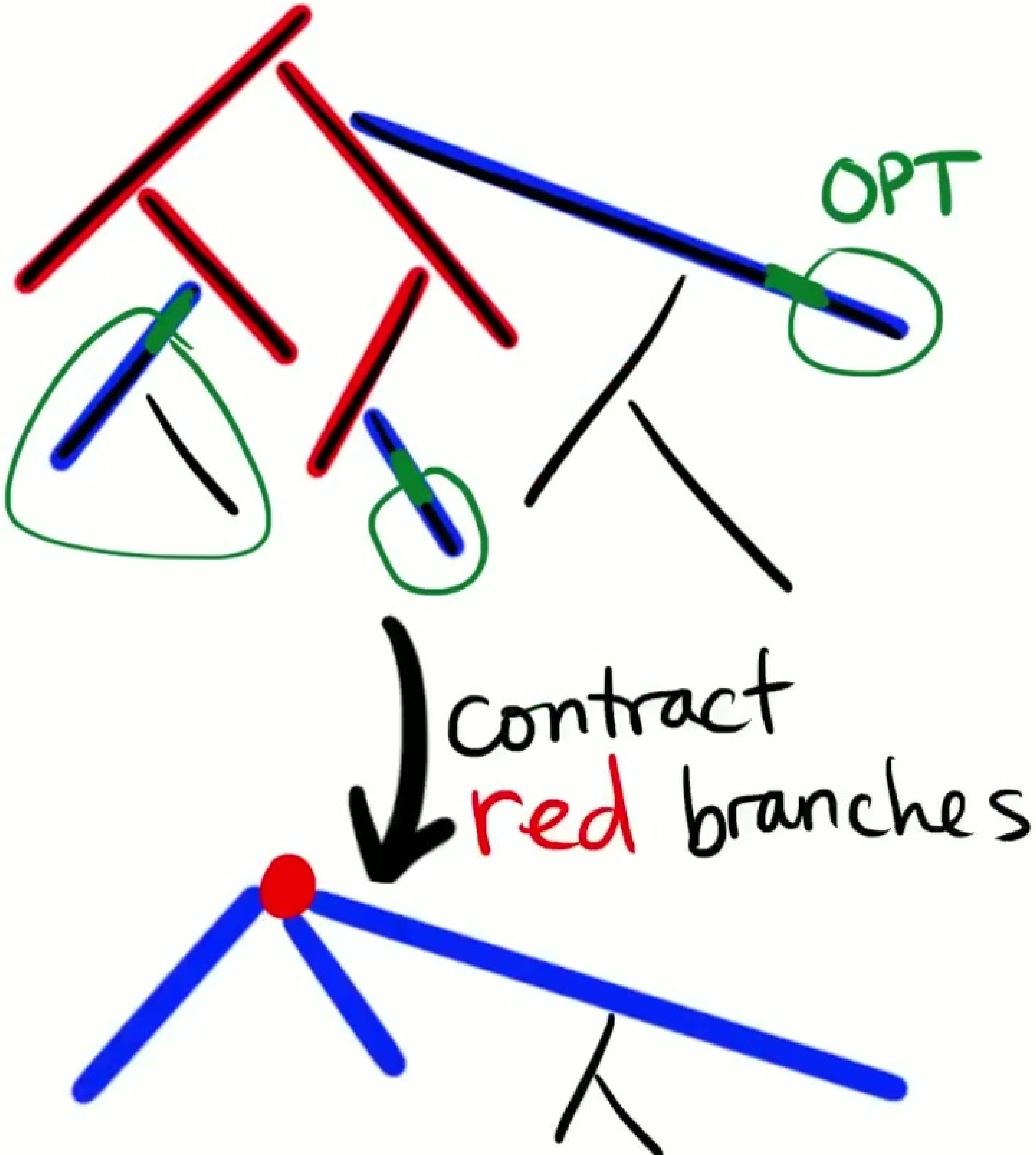

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Reduce to incomparable: Dynamic program on subtrees [GLL18]

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Faster min k-cut on a weighted graph?

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Deterministic n^k time?