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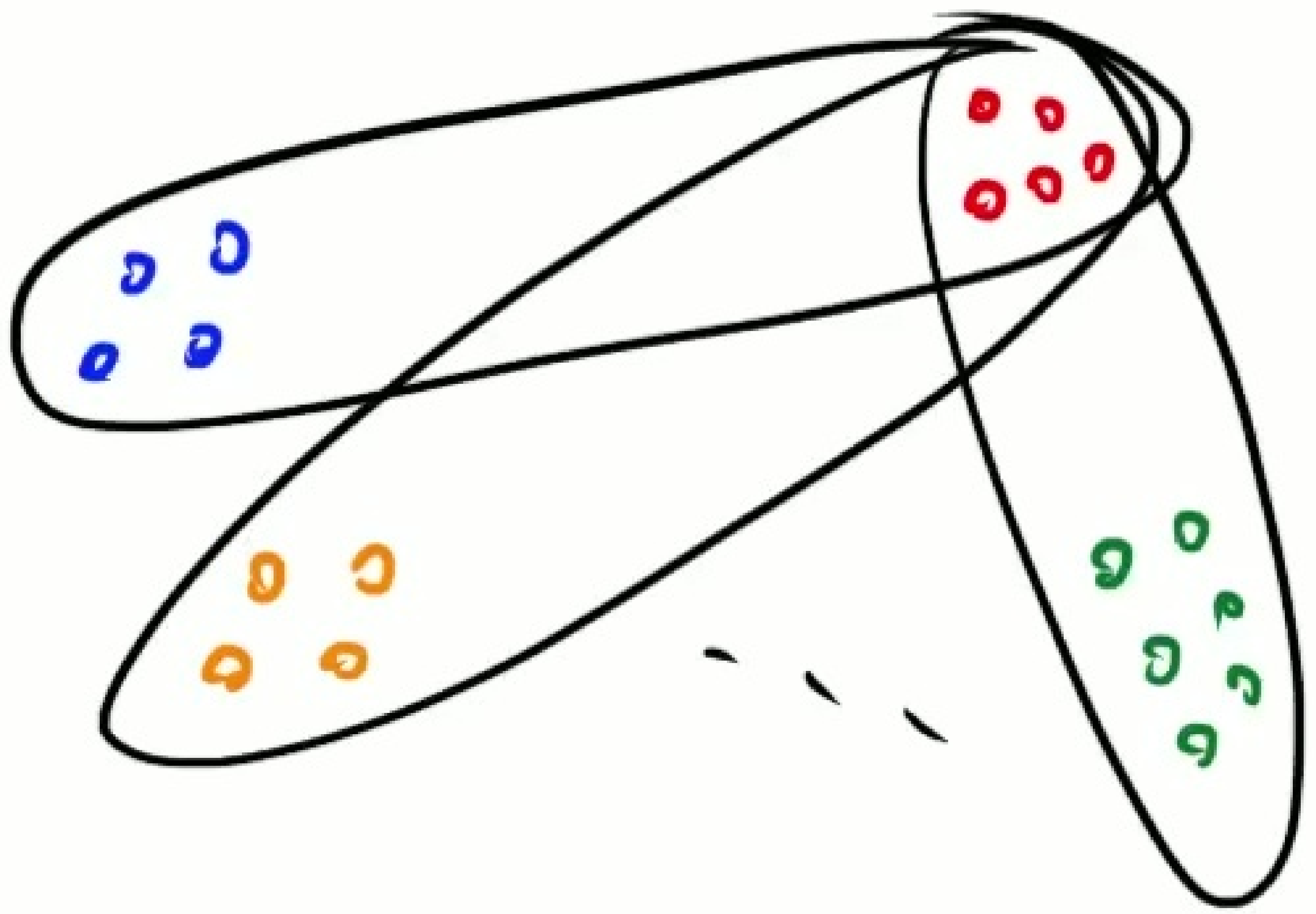

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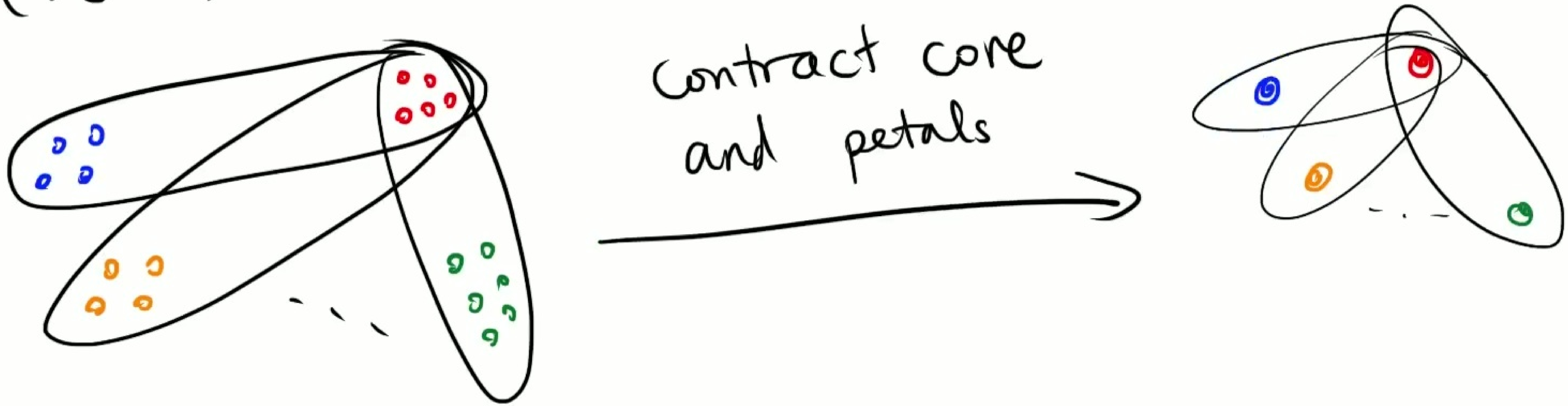

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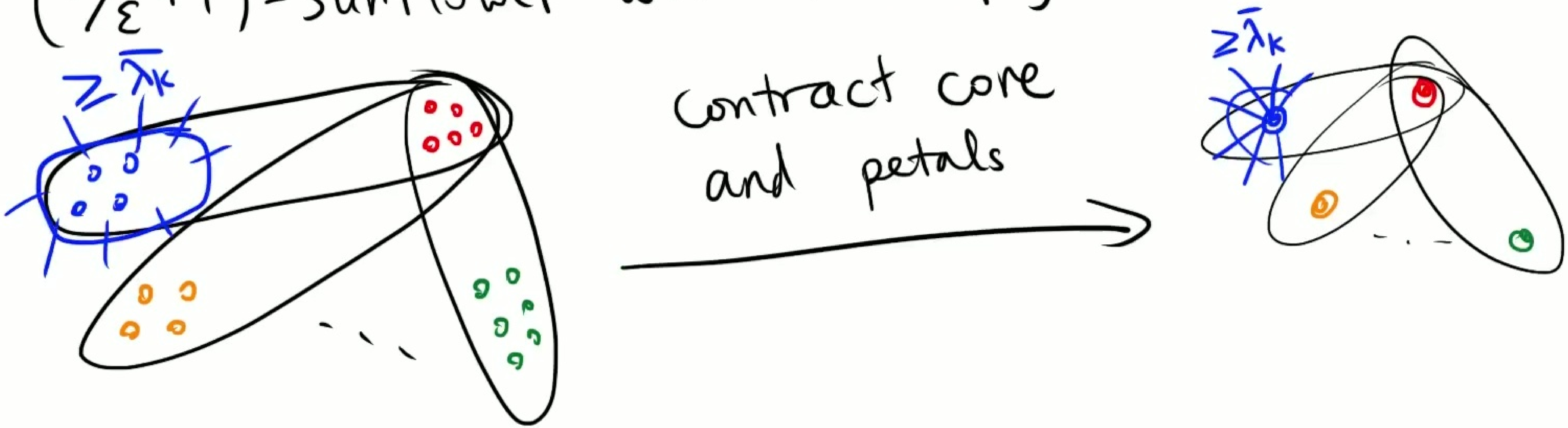

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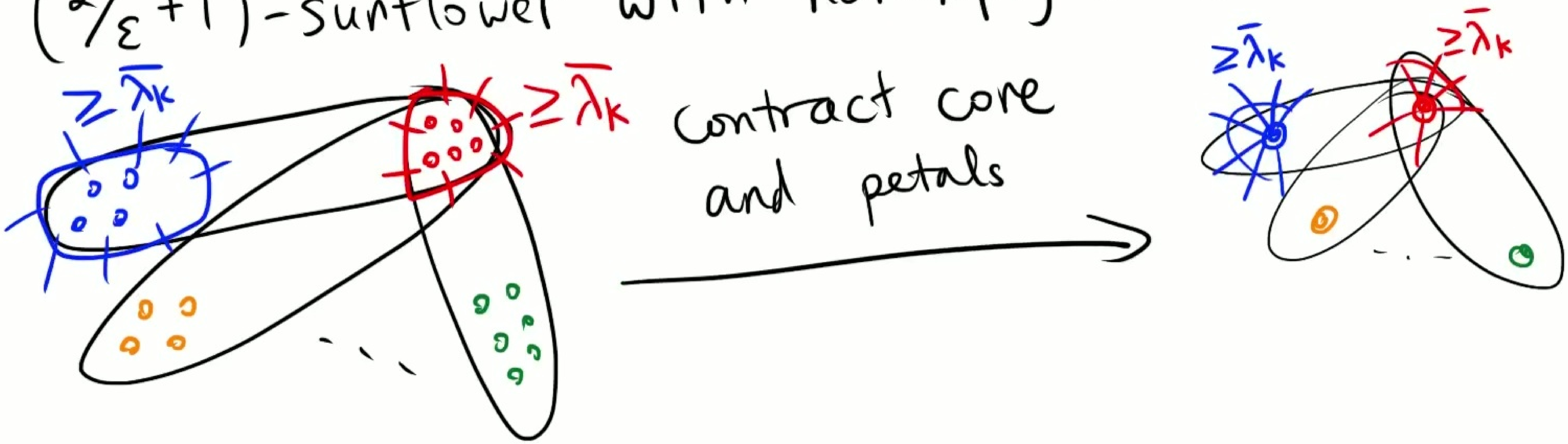

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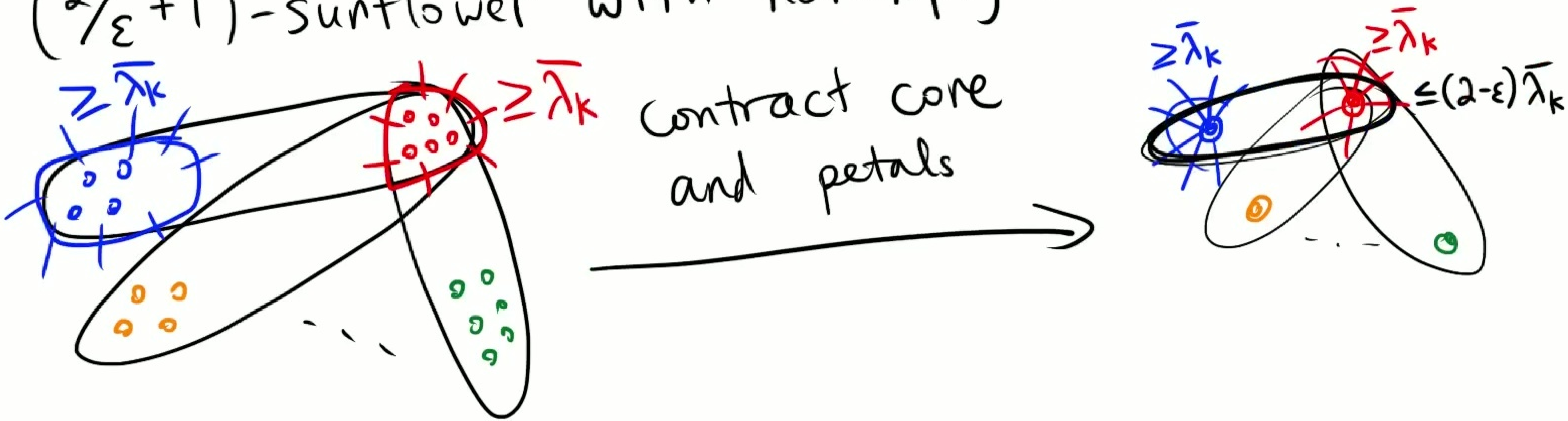

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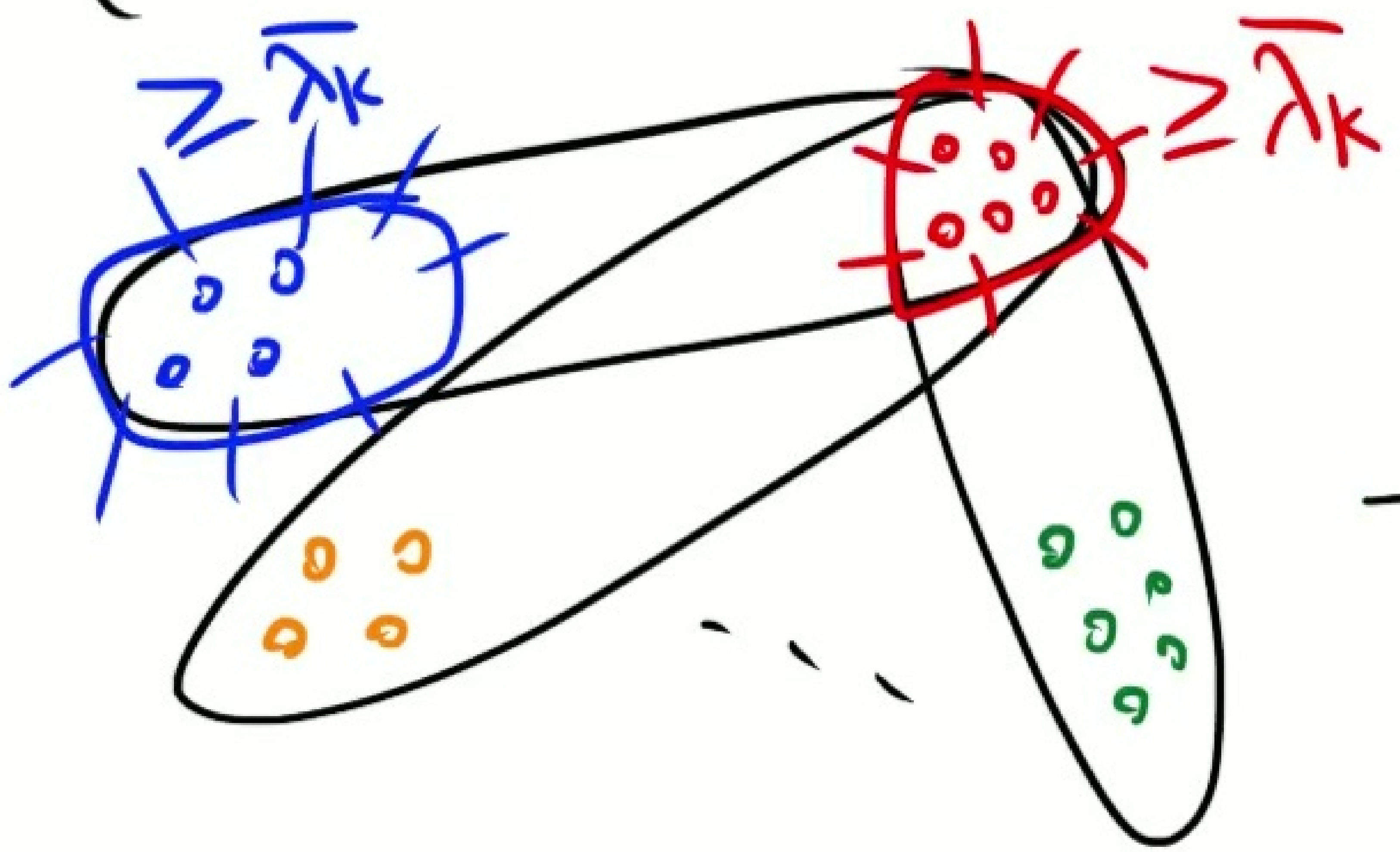
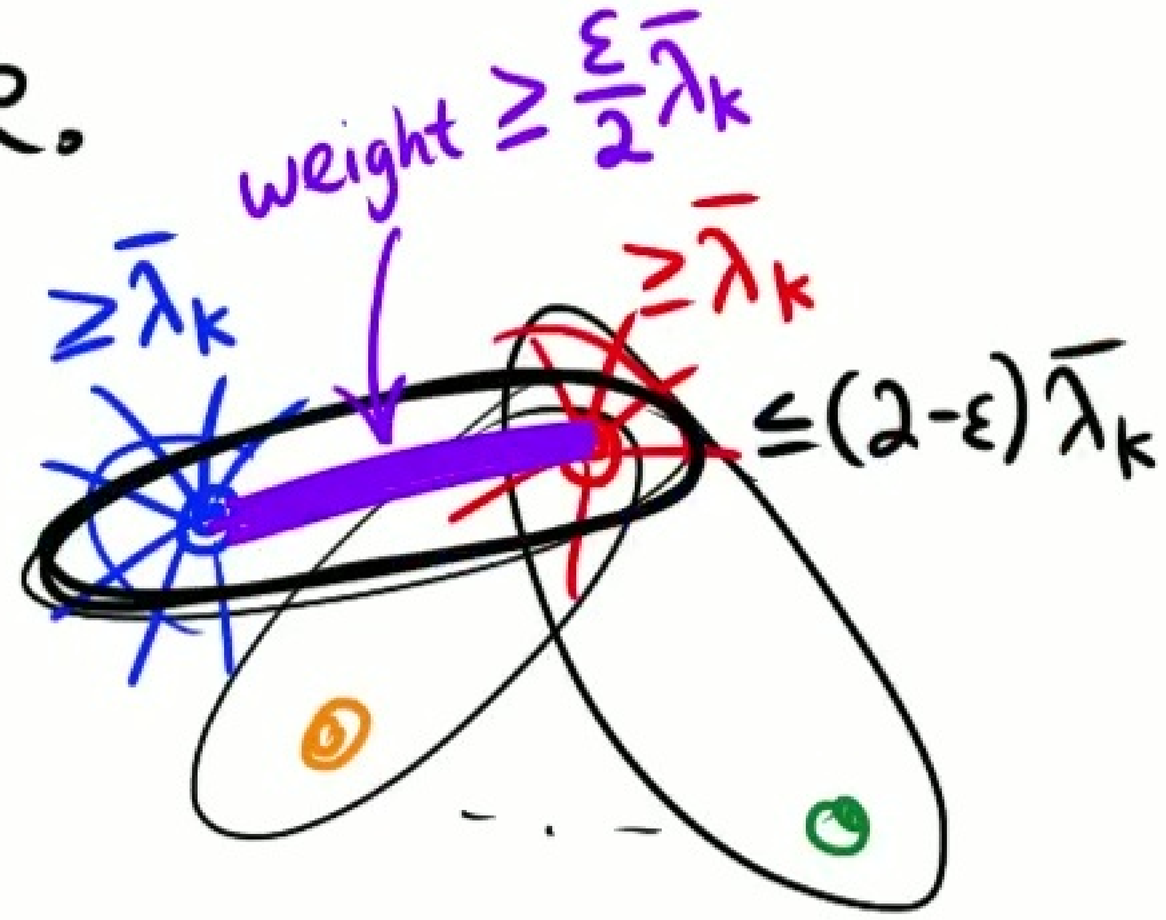


contract core and petals



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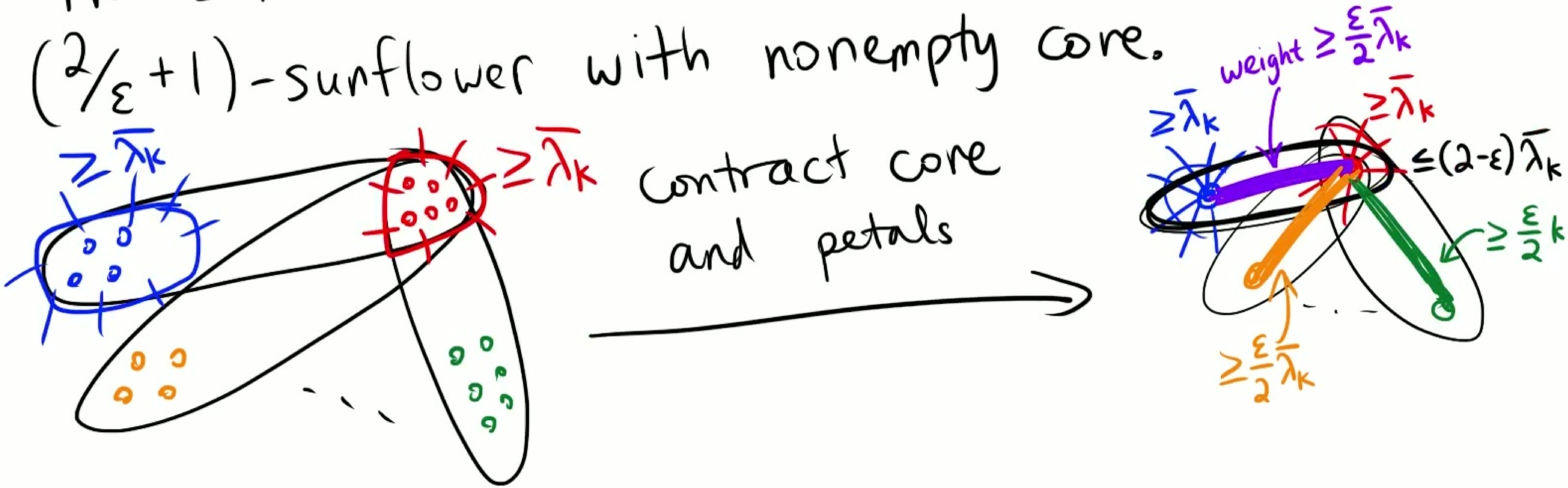

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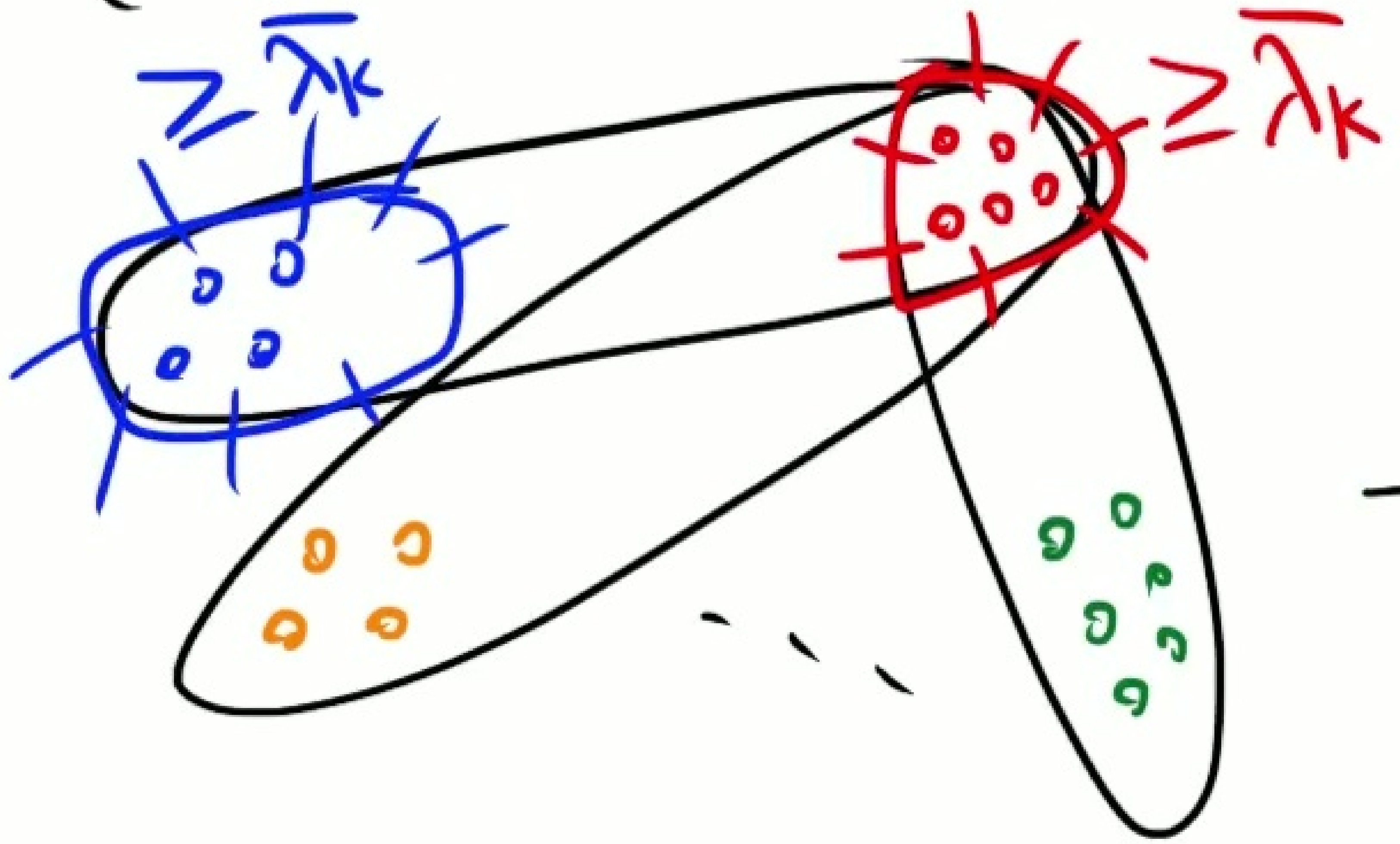

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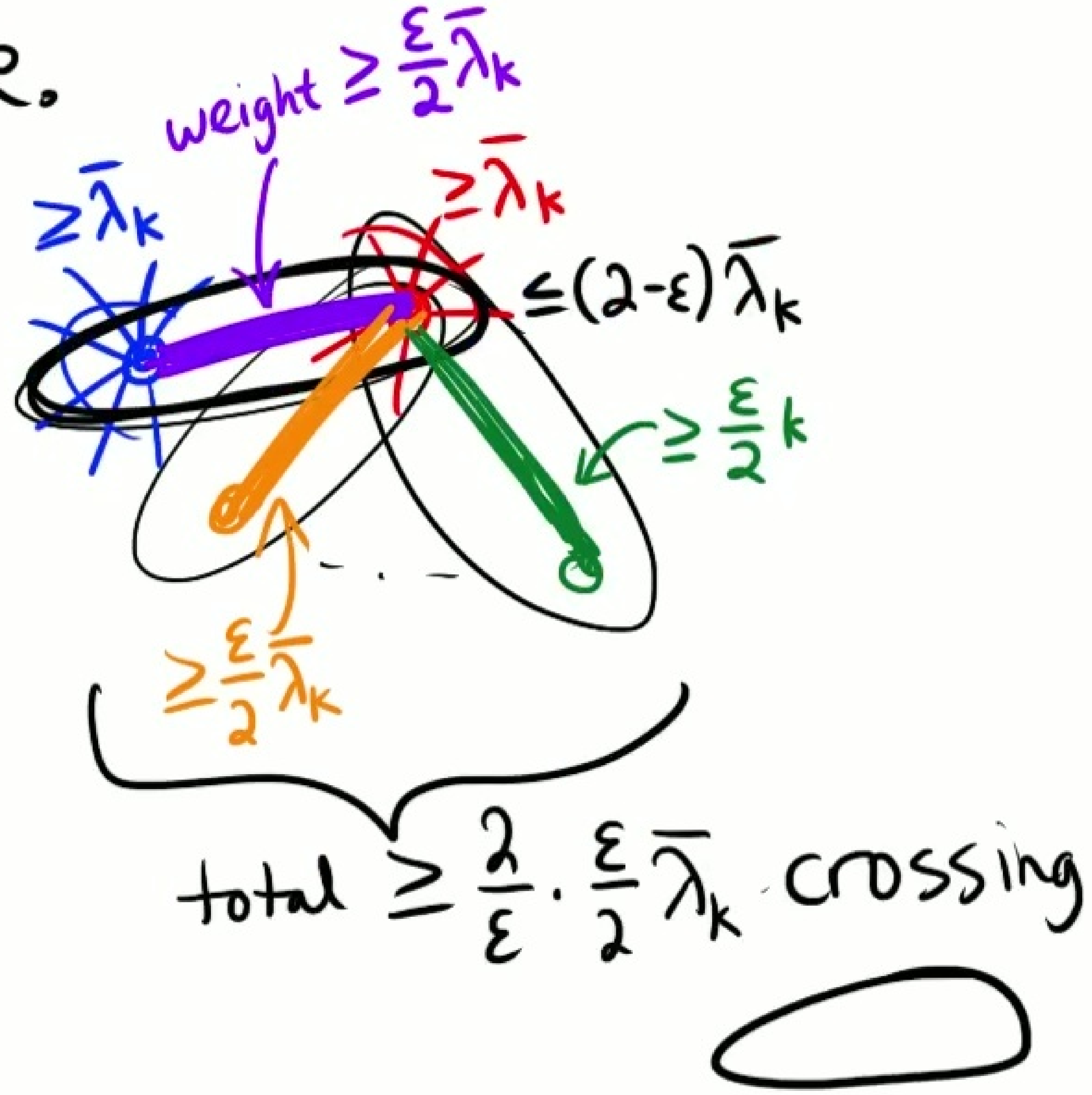

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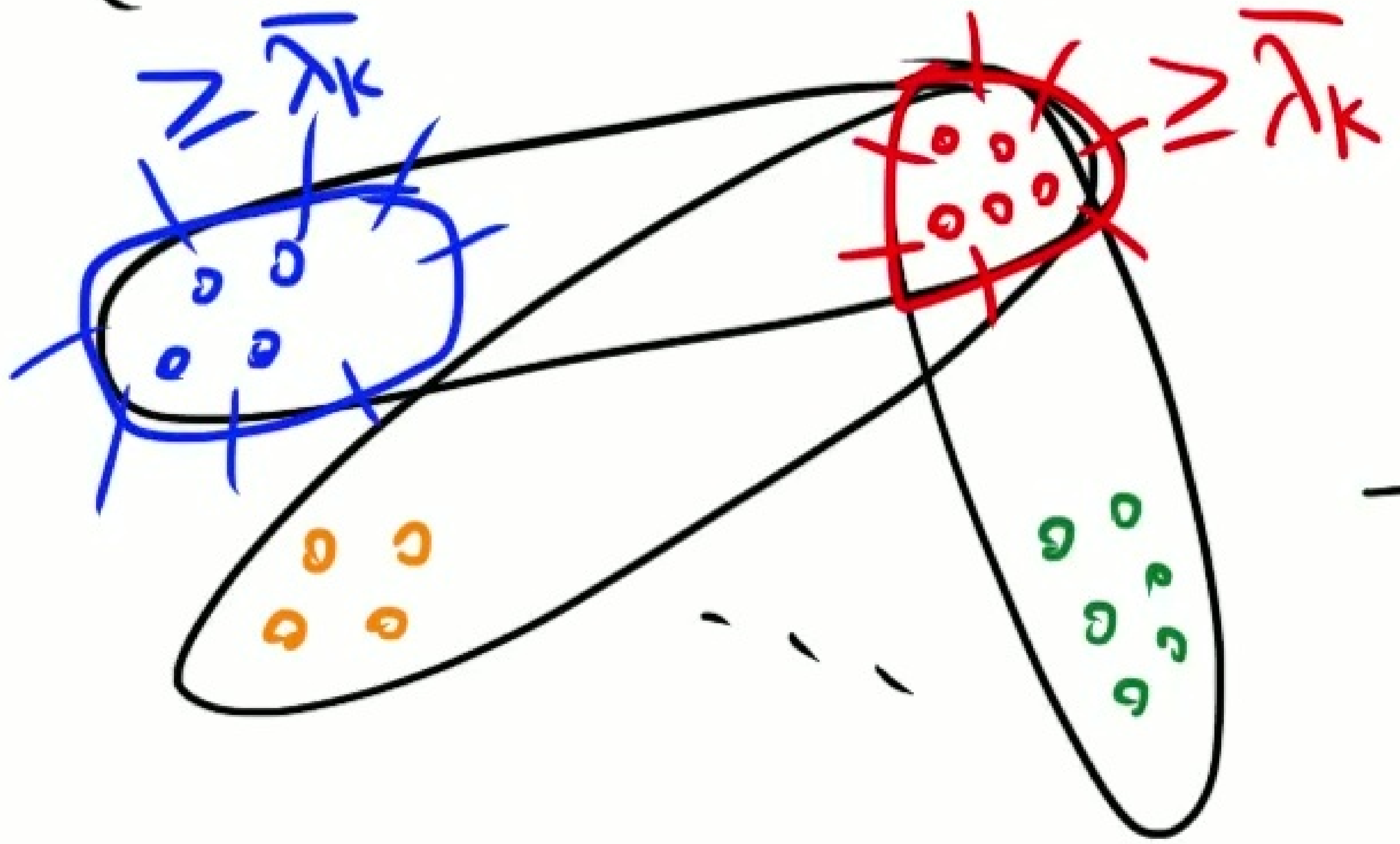

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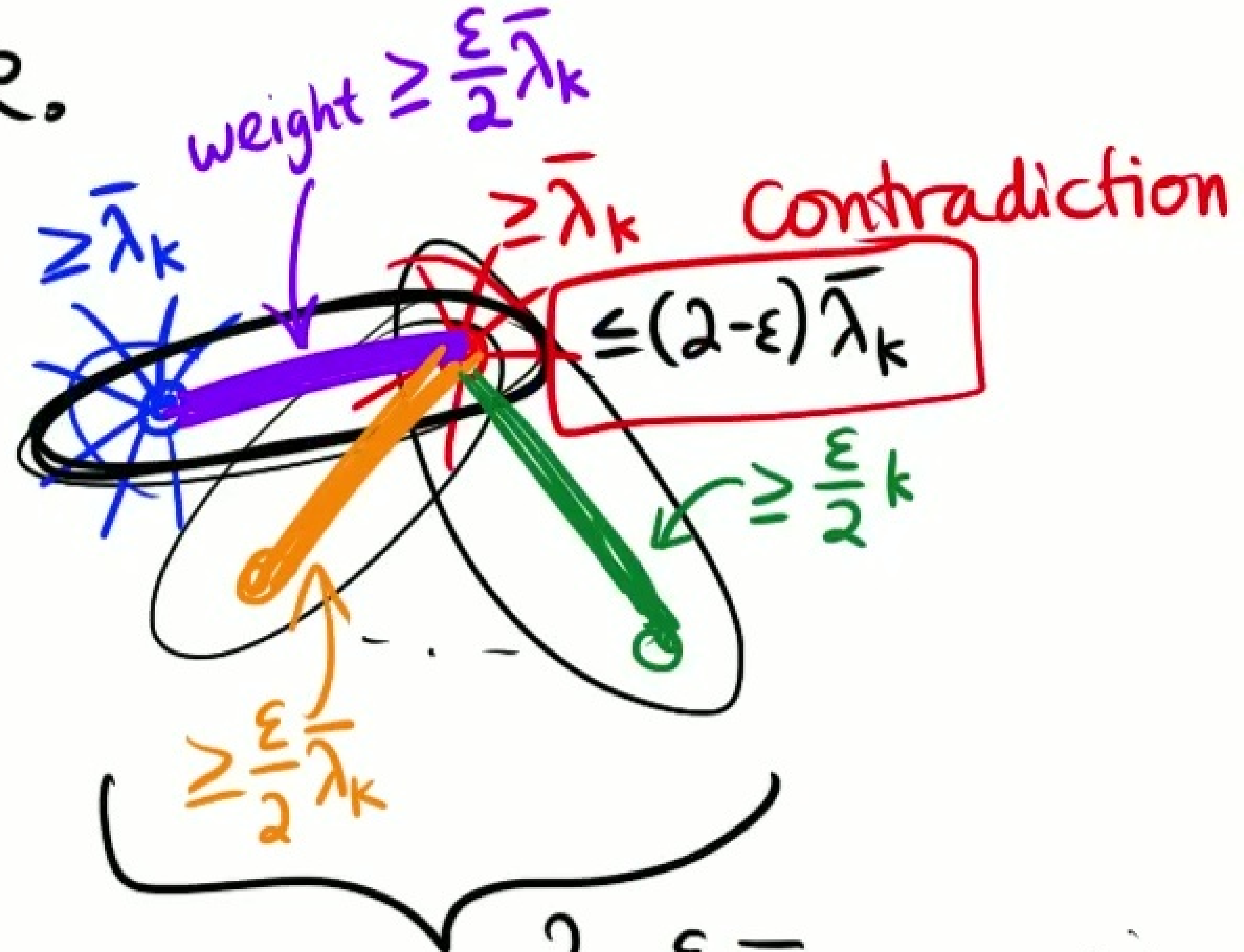

total $\geq \frac{2}{\varepsilon} \cdot \frac{\varepsilon}{2}\bar{\lambda}_k$ crossing

extremal theorem

Thm: suppose a set system on n elements satisfies

① no $k/2$ cuts intersect in $\geq k$ regions

② no r-sunflower

Then, $\#\text{sets} \leq O_{k,r}(n)$

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Base case: all sets size $\leq 5k$. Apply Sunflower Lemma

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Base case: all sets size $\leq 5k$. Apply Sunflower Lemma

Base case: all sets size $\geq n-5k$. Apply Sunflower Lemma

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Base cases: $\leq O(n)$ sets

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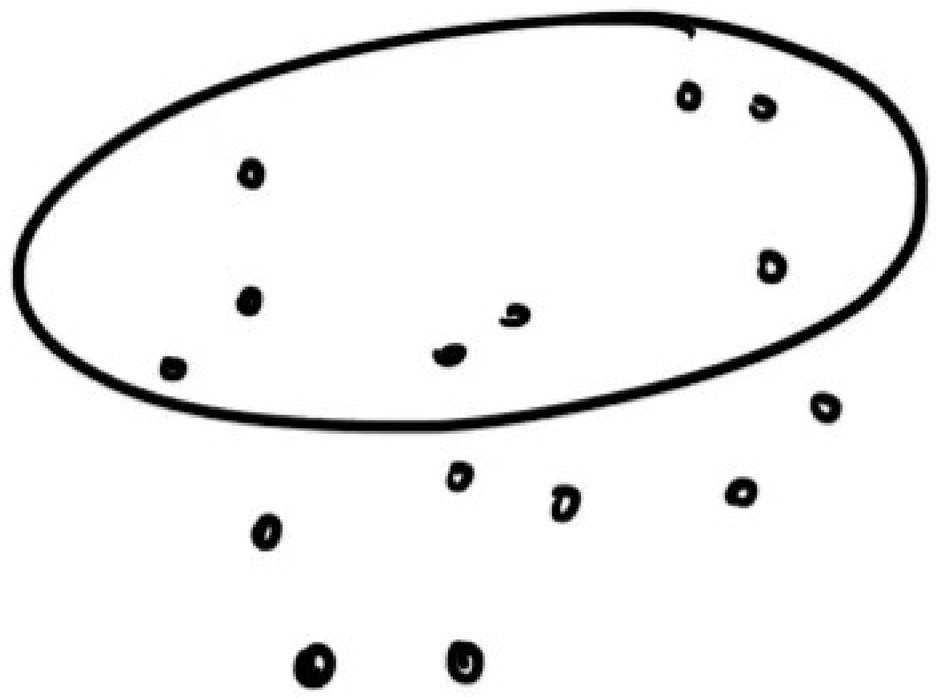
Inductive step: $\exists$ set size $\in [5k, n-5k]$.



extremal theorem

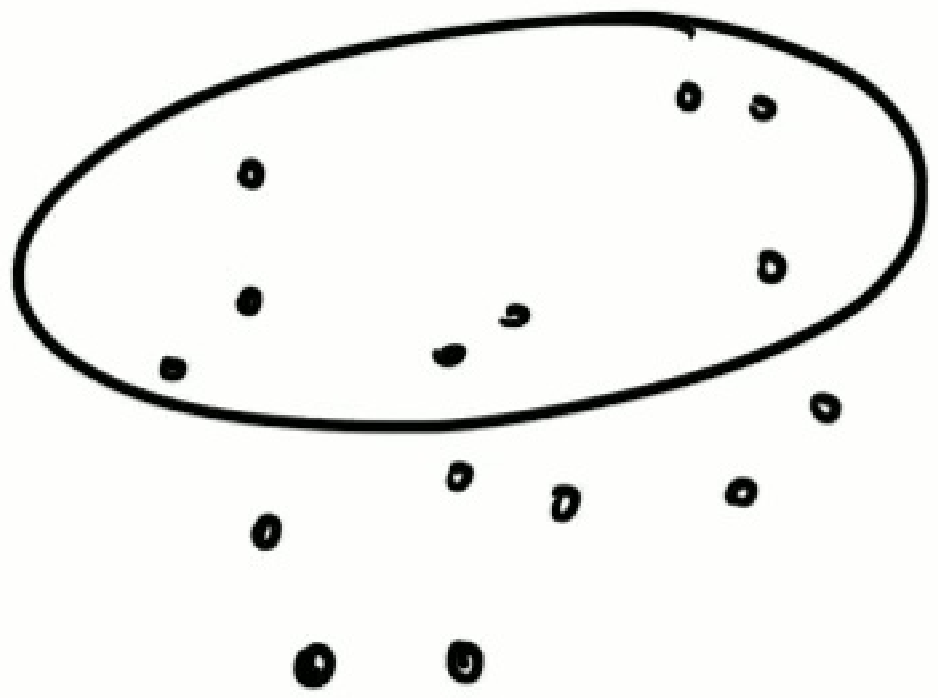
Thm: suppose a set system on n elements satisfies

1. no $k/2$ cuts intersect in $\geq k$ regions
2. no r-sunflower

Then, #sets $\leq O_{k,r}(n)$

Inductive step: $\exists$ set size $\in [5k, n-5k]$.

while can add set to add ≥ 2 regions, do so.



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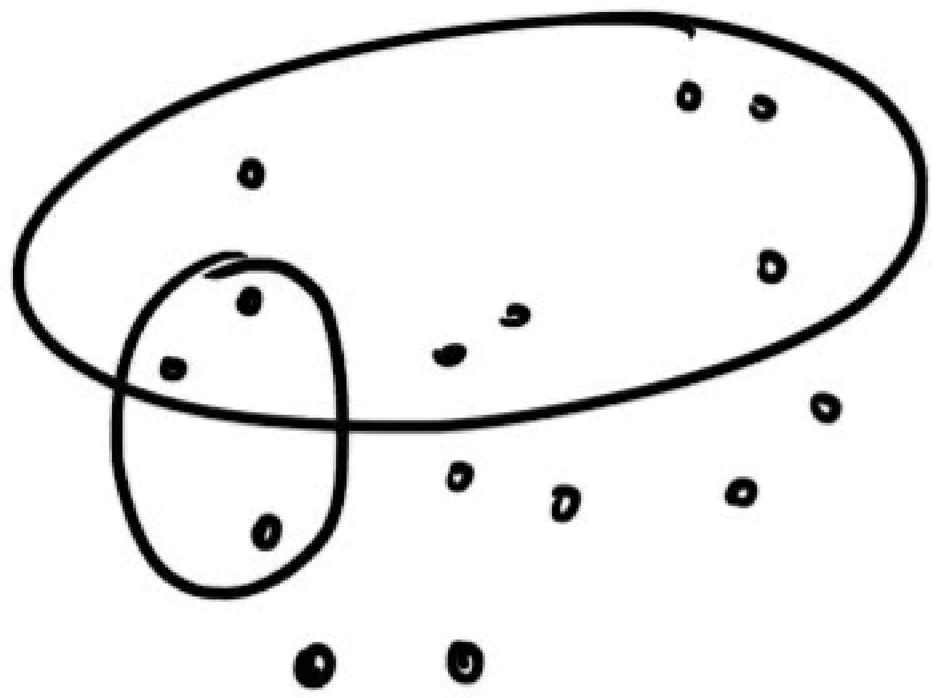

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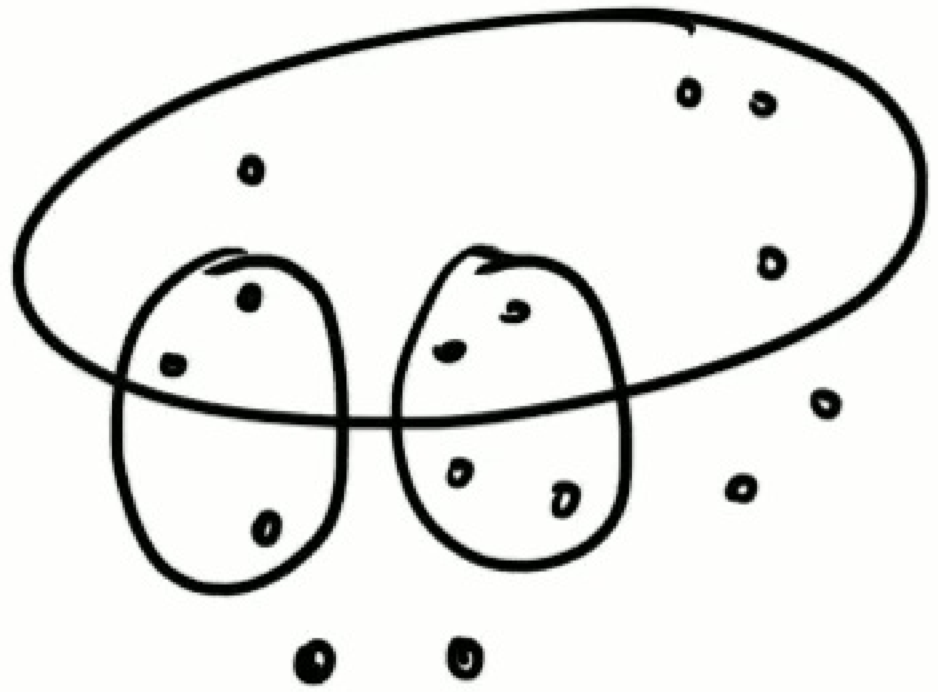

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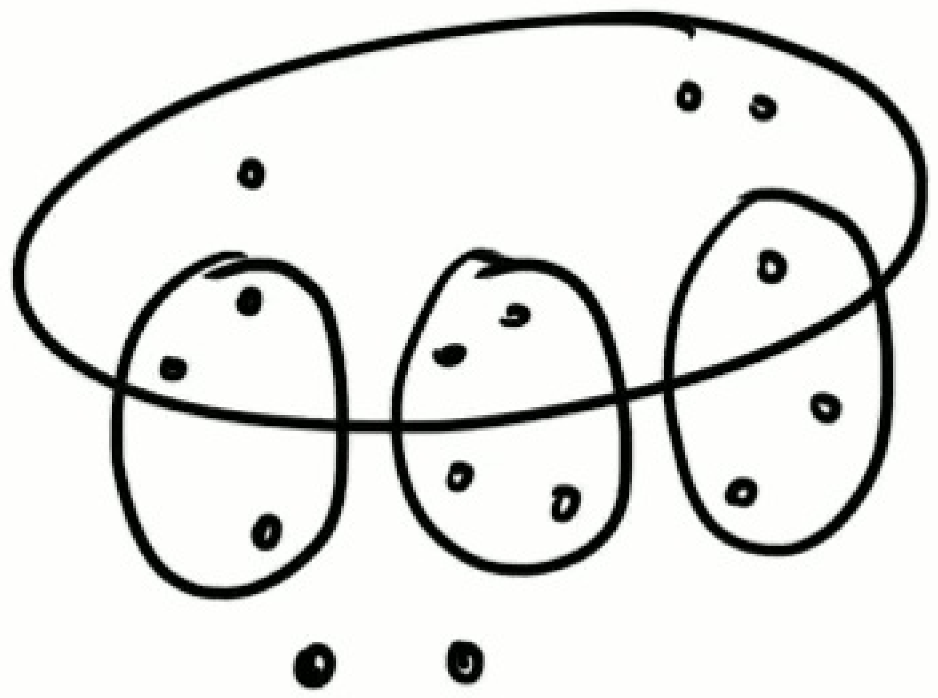

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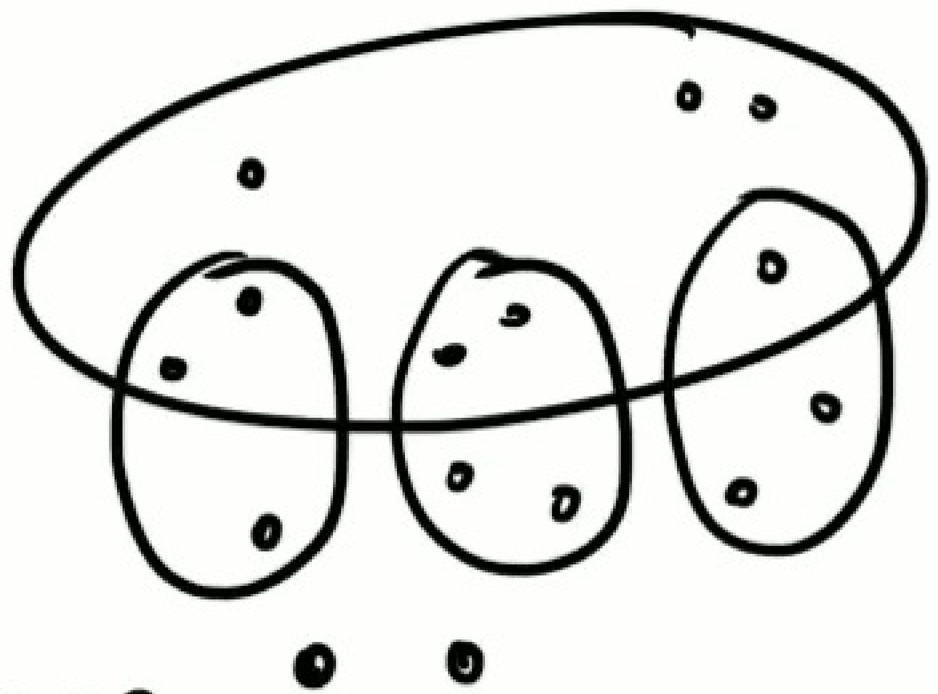

By ②, must stop at $< k$ regions

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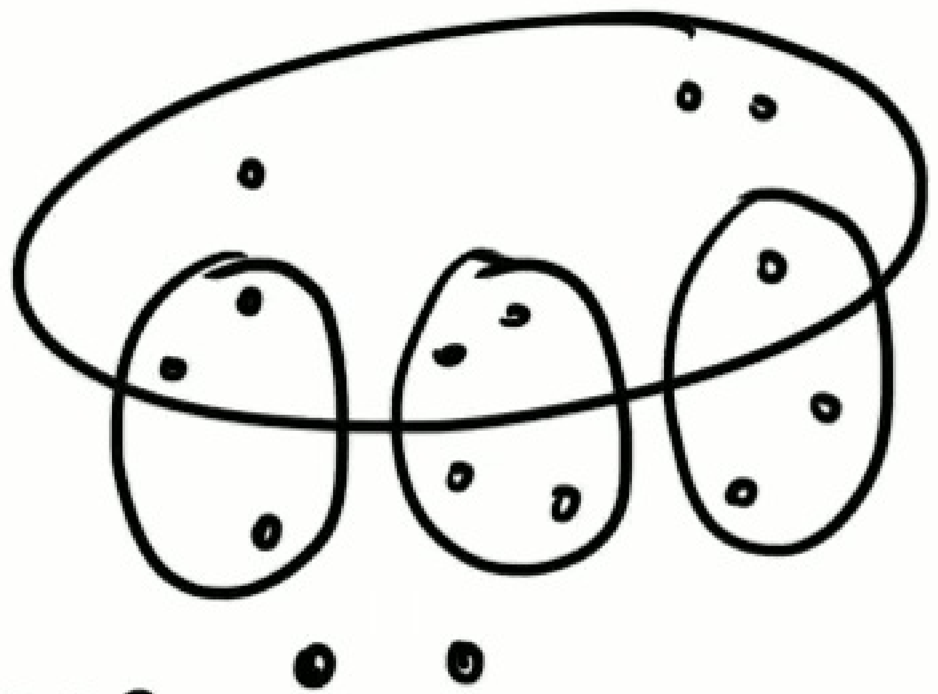

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Every remaining set can cut a region inside or one outside, but not both

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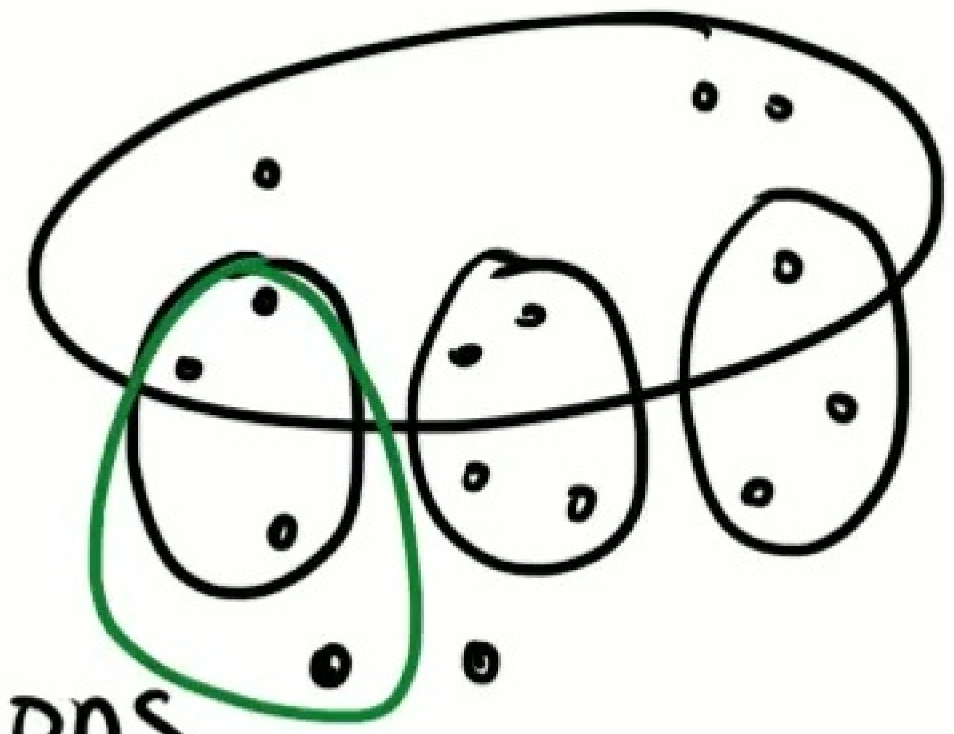

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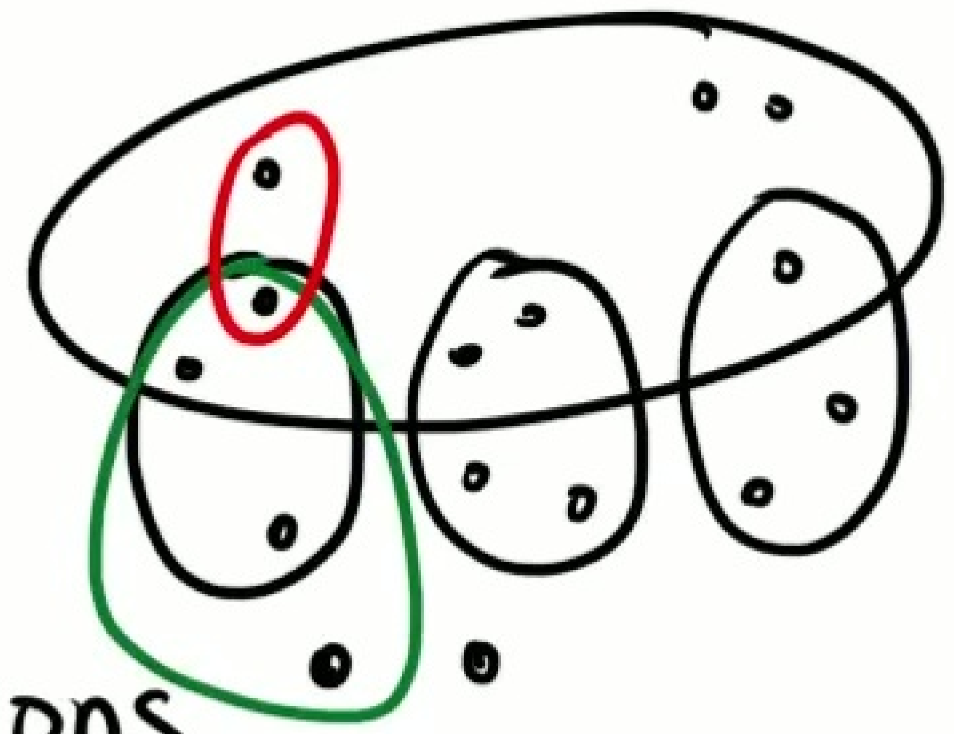

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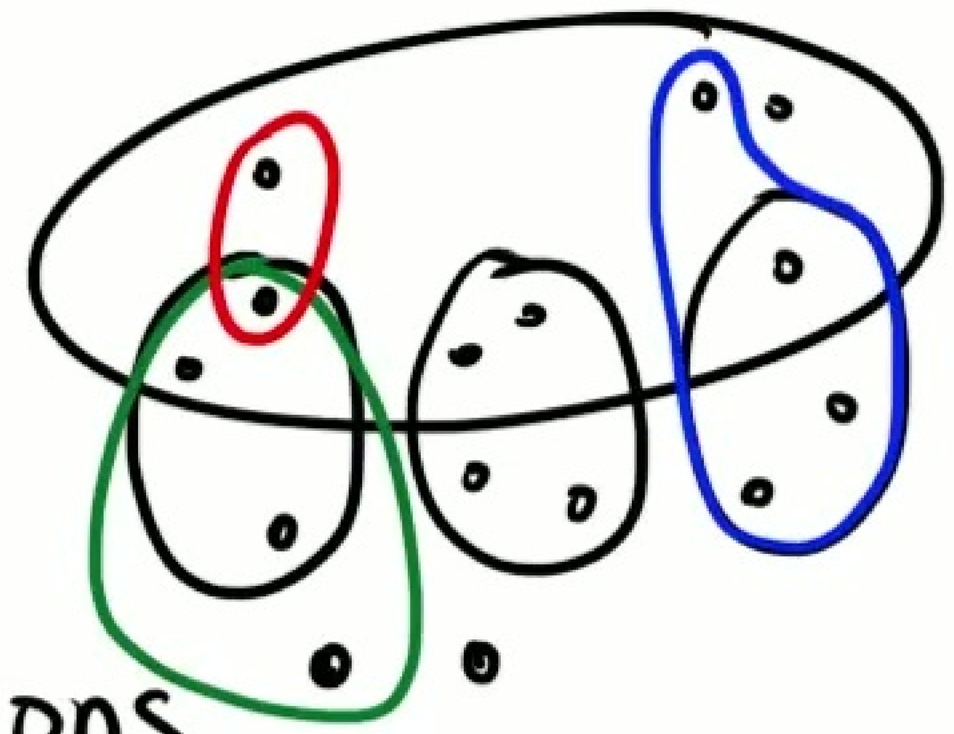

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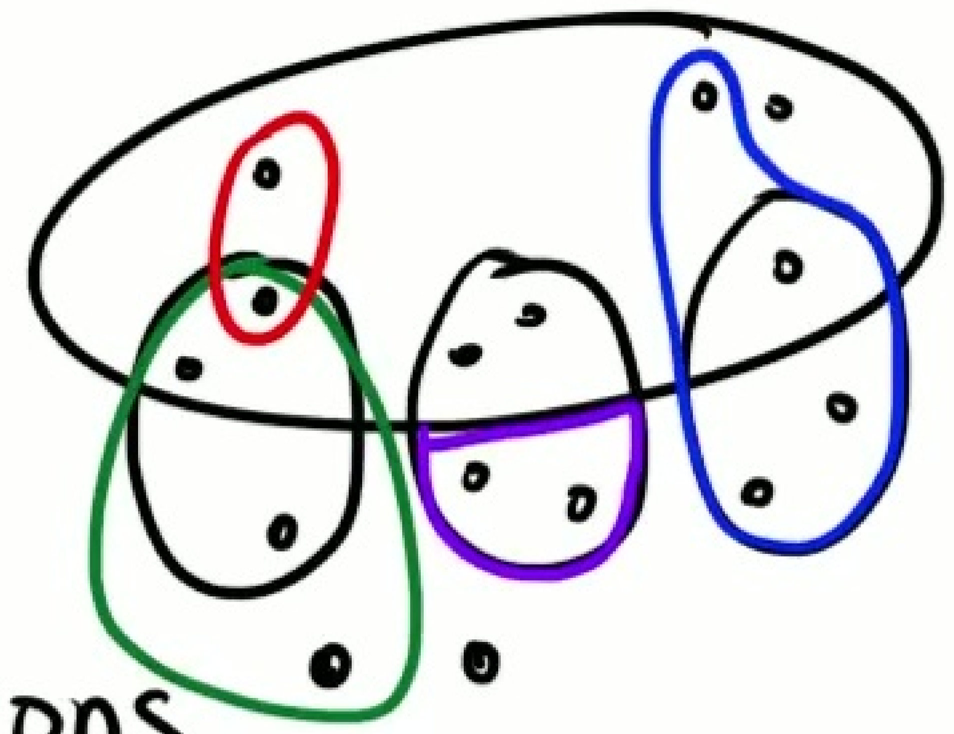

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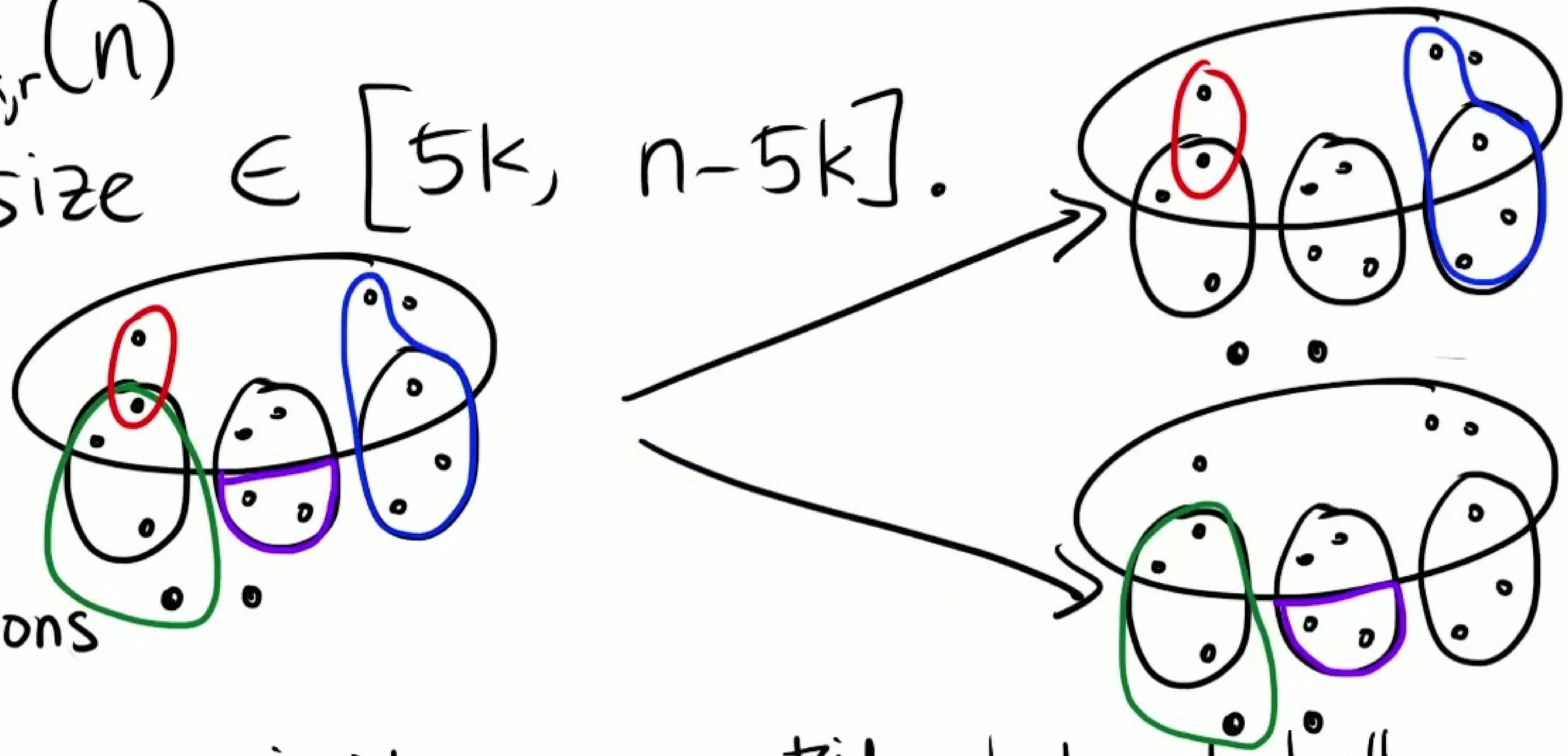

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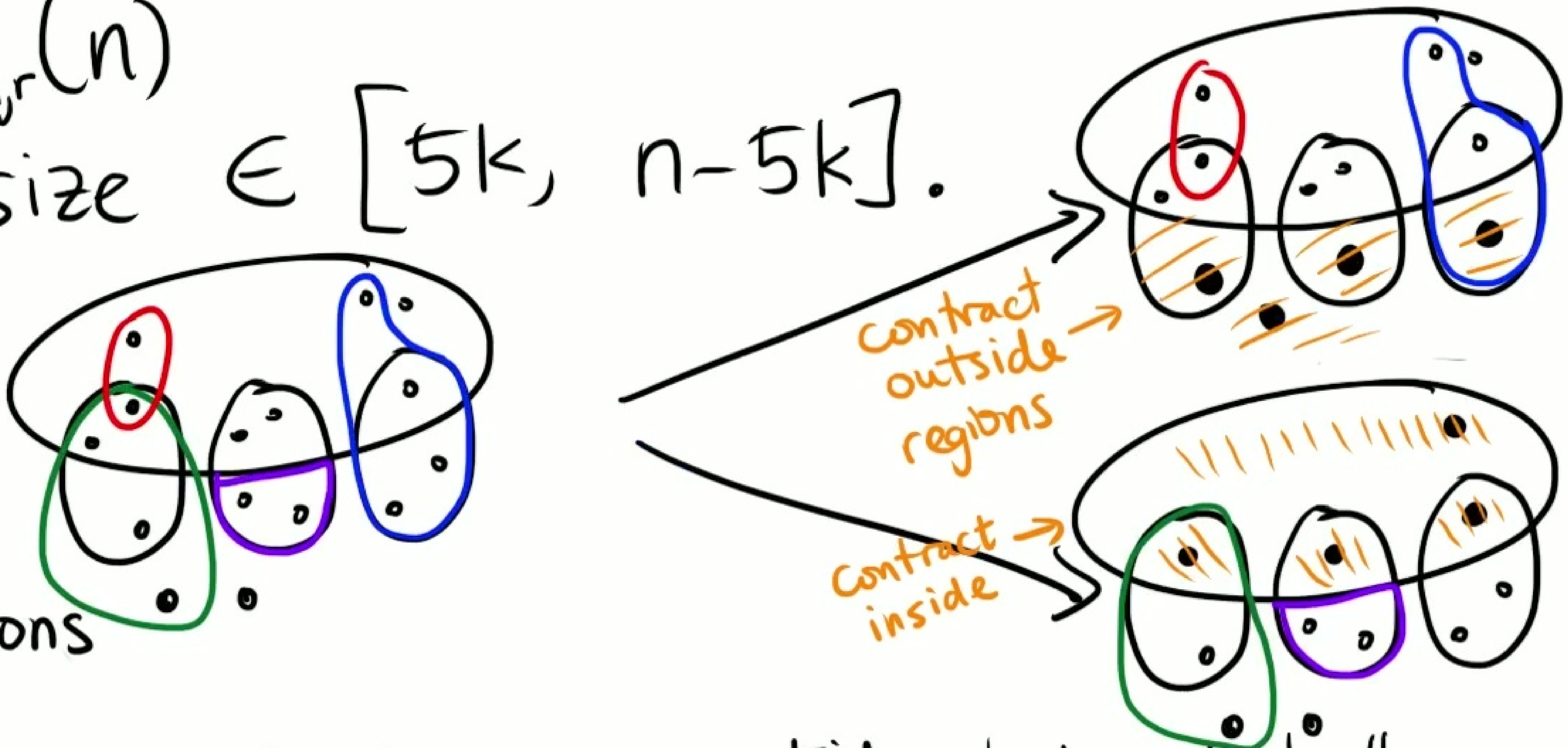

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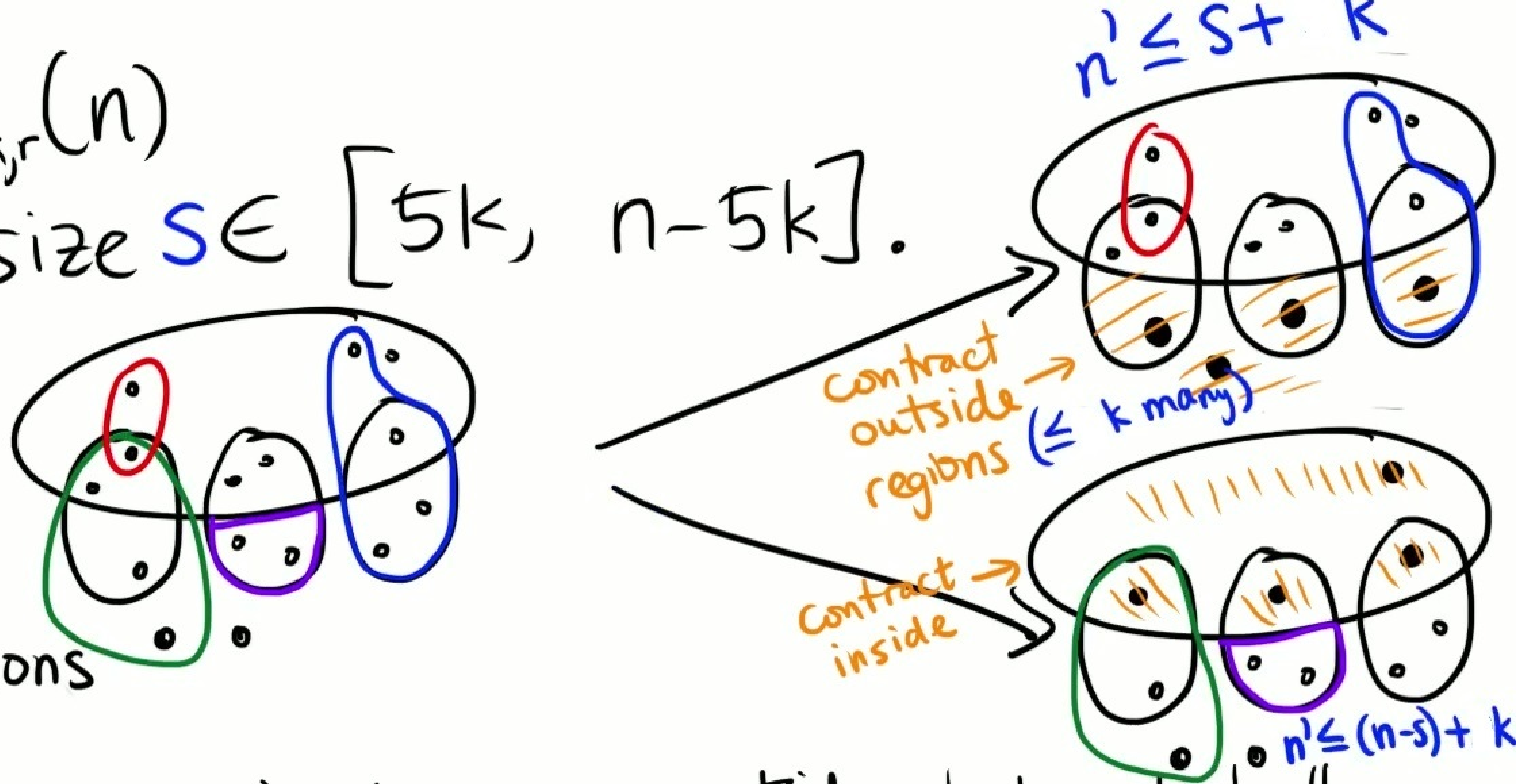

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$$\boxed{f(n) \leq f(s+k) + f(n-s+k)}$$

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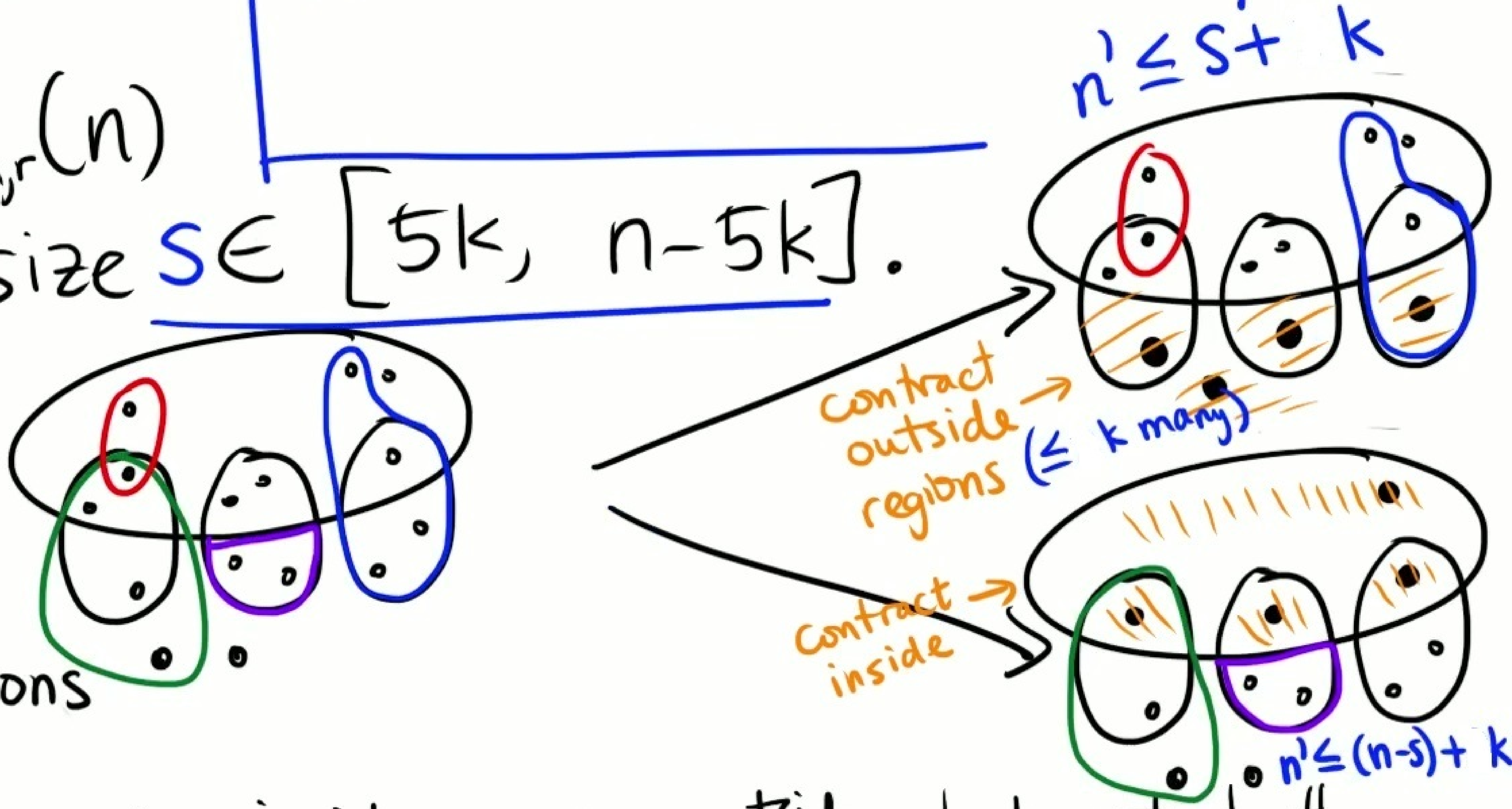

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$$\boxed{\begin{array}{l} f(n) \leq f(\underbrace{s+k}_{\leq n-5k+k<n}) + f(\underbrace{n-s+k}_{<n}) \\ \text{solves to } f(n)=O(n). \end{array}}$$

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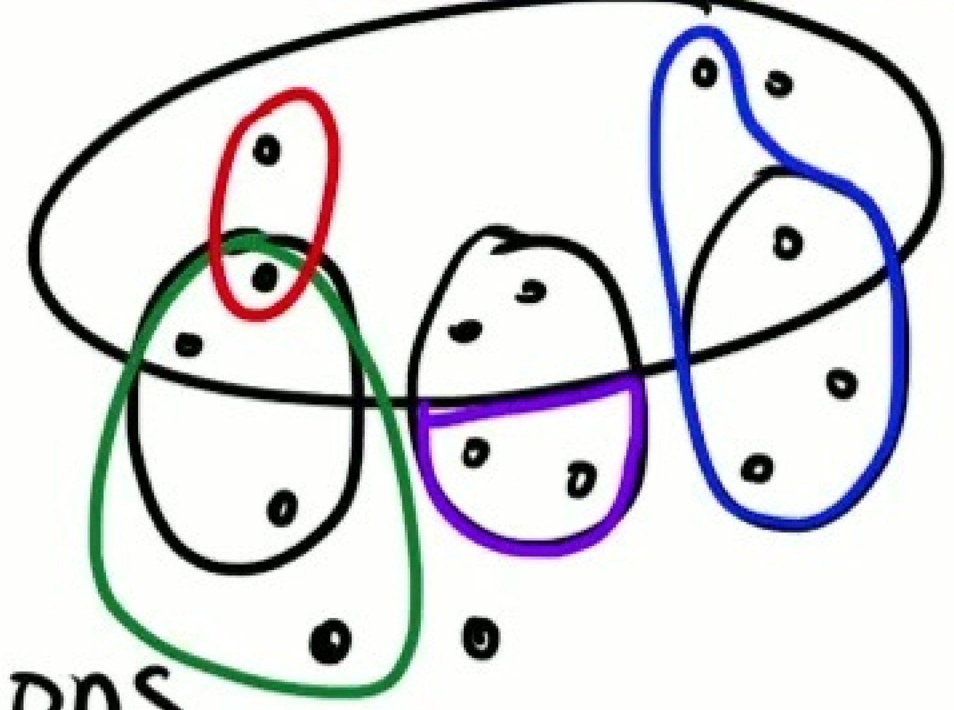

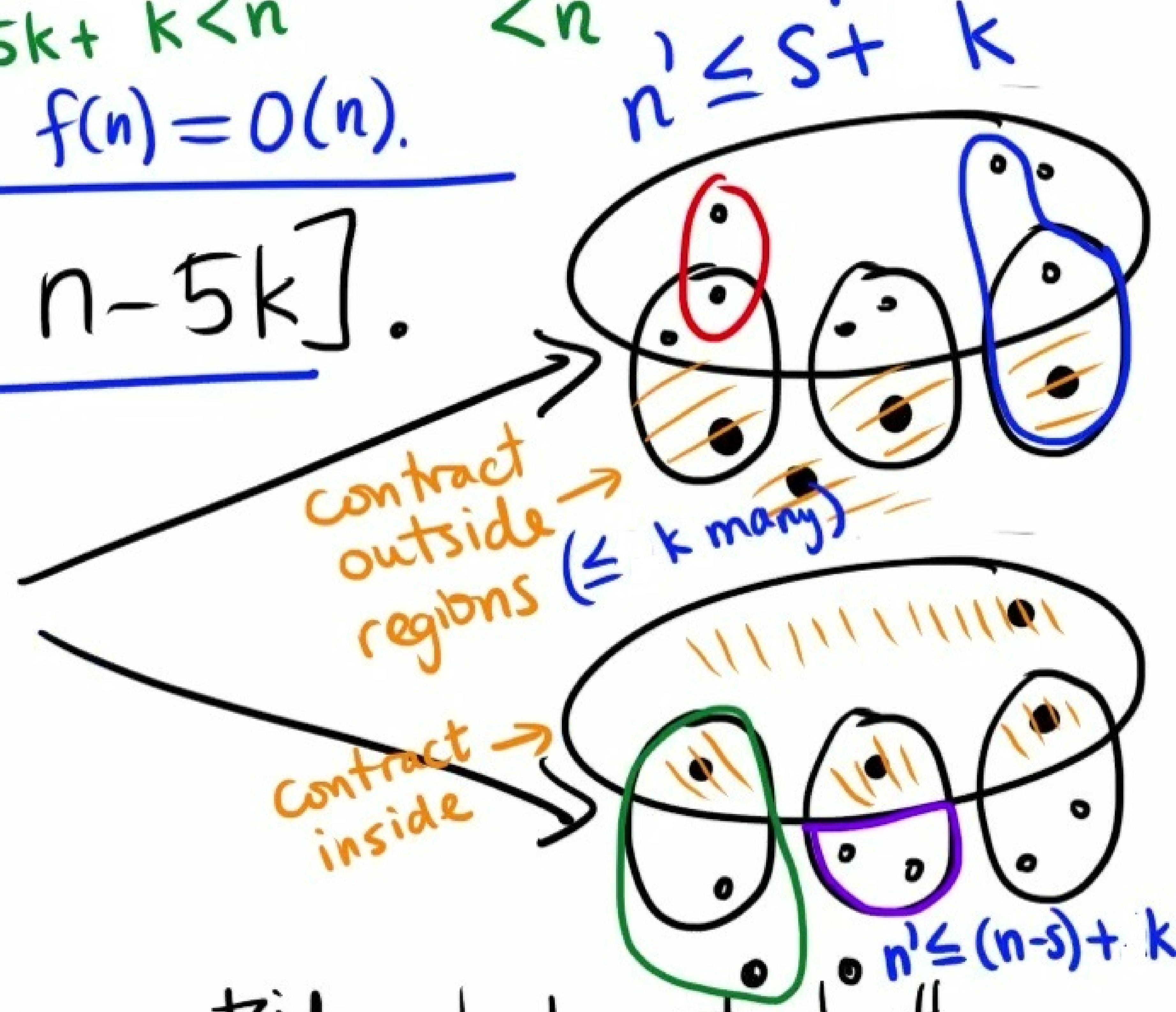

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